WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2021

MTMACOR03T-MATHEMATICS (CC3)

REAL ANALYSIS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Show that for every real number x, there is a positive integer n such that $n > x$.

(b) Find the derived set of the set $S = \left\{(-1)^m + \frac{1}{n}\ ;\ m, n \in \mathbb{N}\right\}$, where $\mathbb{N}$ denotes the set of natural numbers.

(c) For a set S in $\mathbb{R}$, show that the interior S° of S is the largest open contained in $\mathbb{R}$.

(d) For each $x \in (0, 2)$, let $I_x = \left(\frac{x}{2}, \frac{x+2}{2}\right)$. Show that the family $\mathcal{G} = \{I_x : x \in (0, 2)\}$ is an open cover of the set $S = \{x \in \mathbb{R} : 0 < x < 2\}$. Show that no finite subfamily of $\mathcal{G}$ can cover S.

(e) Prove that a convergent sequence is bounded.

(f) Give an example of an unbounded set having exactly one limit point.

(g) If a sequence $(a_n)_n$ of positive real numbers converges to zero, then prove that the sequence has a subsequence $(a_{n_k})_k$ such that the series $\sum\limits_{k=1}^{\infty} a_{n_k}$ is convergent.

(h) Use comparison test (limit form) to test the convergence of the series

$$\frac{1}{\sqrt{1\cdot2}} + \frac{1}{\sqrt{2\cdot3}} + \frac{1}{\sqrt{3\cdot4}} + \cdots\cdots$$

(i) Examine whether the following series converges:

$$1 - \frac{2^2}{2!} + \frac{3^3}{3!} - \frac{4^4}{4!} + \cdots\cdots$$

2. (a) Find $\sup A^c$ and $\inf A^c$, where $A = \{x \in \mathbb{R} : x^2 - x - 12 \geq 0\}$. 3

(b) Let S be a nonempty subset of $\mathbb{R}$. If S is bounded above, then prove that the set $T = \{-x\ :\ x \in S\}$ is bounded below and $\inf T = -\sup S$. 3

(c) If A and B be two nonempty bounded subsets of $\mathbb{R}$ and if $C=\{x+y\ :\ x\in A,\ \ y\in B\}$, then show that $\sup C=\sup A+\sup B$. 2

3. (a) Examine whether the set $\{x\in\mathbb{R}:\ \sin x=0\}$ is countable. 2

(b) Show that an infinite set has a countably infinite subset. 3

(c) What is meant by neighbourhood of a point in $\mathbb{R}$? Check whether the set $\left\{\frac{1}{n}:n\in\mathbb{N}\right\}\cup\{0\}$ is a neighbourhood of 0 or not. 1+2

4. (a) If A be a nonempty bounded subset of $\mathbb{R}$, show that $A'\cap(b,\infty)=\phi$, where $b=\sup A$ and A' is the derived set of A. 2

(b) Prove that every bounded infinite subset of $\mathbb{R}$ has a limit point in $\mathbb{R}$. 3

(c) Show that the set $S=\{x\in\mathbb{R}:|x-1|+|x-2|<3\}$ is an open set. 3

5. (a) Give an example of a bounded set in $\mathbb{R}$ which is not compact and a closed set in $\mathbb{R}$, which is not compact (with justifications). 2+2

(b) Define closed set in $\mathbb{R}$, in terms of its limit points. Hence, show that 1+1+2

(i) intersection of arbitrary collection of closed sets in $\mathbb{R}$ is also closed in $\mathbb{R}$, and

(ii) there is no proper nonempty subset of $\mathbb{R}$ which is both open and closed.

6. (a) State sequential definition of compact sets in $\mathbb{R}$. Hence, show that a set in $\mathbb{R}$ is compact if and only if it is closed and bounded. 1+3

(b) Let E be a closed and bounded subset of $\mathbb{R}$. Prove that every open cover of E has a finite subcover. 4

7. (a) Define convergence of a sequence in $\mathbb{R}$. 1

(b) Prove that a sequence in $\mathbb{R}$ may converge to at most one limit in $\mathbb{R}$. 2

(c) Let $(x_n)_n$ be a sequence of real numbers and let $x\in\mathbb{R}$. If $(a_n)_n$ is a sequence of positive real numbers with $\lim a_n=0$ and if for some constant $c>0$, there exists $m\in\mathbb{N}$ such that $|x_n-x|\leq c\,a_n$, for all $n\geq m$ then, from the definition of convergence of sequence, prove that $\lim x_n=x$. 2

(d) Using the result stated in (c) above, prove that $\lim\,(x^{1/n})=1$ for any real number $x>0$. 3

8. (a) If a sequence $(a_n)_n$ converges to zero and also if the sequence $(b_n)_n$ is bounded, then show that the sequence $(a_nb_n)_n$ converges to zero. 3

(b) Let $A(\subseteq\mathbb{R})$ be dense in $\mathbb{R}$. Then, for every $a\in\mathbb{R}$, prove that there is a sequence $(a_n)_n$ of elements in A, which converges to a. 2

(c) Prove that the sequence $\{x_n\}$ where $x_1 = 1$ and $x_{n+1} = \frac{4+3x_n}{3+2x_n}$, $\forall\, n \geq 1$, is convergent and converges to $\sqrt{2}$. 3

9. (a) Let $(x_n)_n$ be a sequence of real numbers defined by $x_1 = \frac{1}{3}$, $x_{2n} = \frac{1}{3} x_{2n-1}$ and $x_{2n+1} = \frac{1}{3} + x_{2n}$ for $n = 1, 2, 3, \cdots\cdots$. Find $\liminf x_n$ and $\limsup x_n$. 3

(b) Prove that every Cauchy sequence in $\mathbb{R}$ is convergent. 2

(c) Show that the ratio test is not suitable for arriving at any conclusion about convergence of the series 3

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots\cdots$$

Examine the convergence of this series by applying root test.

10.(a) Use Cauchy's integral test to show that the series $\sum \frac{1}{n(\log n)^p}$, $p > 0$, converges for $p > 1$ and diverges for $p \leq 1$. 4

(b) Let $a \in \mathbb{R}$ with $a > 0$. Show that the series 4

$$\frac{1}{(1+a)^p} - \frac{1}{(2+a)^p} + \frac{1}{(3+a)^p} - \cdots\cdots \text{ is}$$

(i) absolutely convergent if $p > 1$, and

(ii) conditionally convergent if $0 < p \leq 1$.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2021

MTMACOR08T-MATHEMATICS (CC8)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Prove that $f:[0,3]\to\mathbb{R}$ defined by $f(x)=x+[x]$ is integrable.

(b) Give an example, with proper justifications, of a discontinuous function which has a primitive.

(c) Show that the integral $\int_0^1 \frac{1}{\sqrt{x}}\sin\frac{1}{x}\,dx$ is absolutely convergent.

(d) Evaluate $\int_0^{\pi/2} \sin^{3/2}\theta\cos^3\theta\,d\theta$, assuming convergence of the given integral.

(e) Examine whether the sequence of functions $\{f_n\}$ converges uniformly on $\mathbb{R}$, where for all $n\in\mathbb{N}$,

$$f_n(x)=\frac{x+nx^2}{n},\quad x\in\mathbb{R}$$

(f) Find the limit function $f(x)$ of the sequence $\{f_n\}$ on [0, 1], where for all $n\in\mathbb{N}$,

$$f_n(x)=\begin{cases} nx \ ; & 0\le x\le \frac{1}{n} \\ 1 \ ; & \frac{1}{n}<x\le 1 \end{cases}$$

Hence, state with reason whether $\{f_n\}$ converges uniformly on [0, 1].

(g) Show that the series $\sum_{n=1}^{\infty}\frac{1}{n^3+n^2x^2}$ is uniformly convergent on $\mathbb{R}$.

(h) Determine the radius of convergence of the power series $\sum_{n=1}^{\infty}(2+(-1)^n)^n x^n$.

(i) Show that the series $\sum_{n=1}^{\infty}\frac{x\sin(n^2x)}{n^2}$ converges to a continuous function on [0, 1].

2. (a) If a function $f:[a,\,b]\longrightarrow\mathbb{R}$ be integrable and $f(x)\geq 0$ for $x\in[a,\,b]$ and there exists a point $c\in[a,\,b]$, such that f is continuous at c with $f(c)>0$, then prove that $\int_a^b f>0$. 4

(b) Let f be continuous on $[a, b]$ and for each α, β, $a\leq\alpha<\beta\leq b$, 4

$$\int_{\alpha}^{\beta} f(x)\,dx=0$$

Prove that f is identically zero on $[a, b]$.

3. (a) If a function $f:[a,\,b]\longrightarrow\mathbb{R}$ be bounded and for every $c\in(a,\,b)$, f is integrable on $[c, b]$, then prove that f is integrable on $[a, b]$. 4

(b) Give an example of a function $f:[0,\,1]\longrightarrow\mathbb{R}$ which is integrable on $[c, 1]$, $0<c<1$ but not integrable on $[0, 1]$. 4

4. (a) Let $f_n: D\to\mathbb{R}$ be bounded functions on $D\subseteq\mathbb{R}$, for all $n\in\mathbb{N}$ so that the sequence of functions $\{f_n\}$ is uniformly convergent to $f: D\to\mathbb{R}$. Show that f is bounded on D. 3

(b) Find the limit function $f(x)$ of the sequence $\{f_n\}$, where for all $n\in\mathbb{N}$, 5

$$f_n(x)=\frac{nx}{1+nx},\quad x\in[0,\,1]$$

Justify that $\{f_n\}$ is not uniformly convergent on $[0, 1]$. Further show that $f(x)$ is Riemann integrable on $[0, 1]$ and

$$\lim_{n\to\infty}\int_0^1 f_n(x)\,dx=\int_0^1 f(x)\,dx$$

5. (a) Let $\{a_n\}$ be a convergent sequence of real numbers and let $\{f_n\}$ be a sequence of functions satisfying 3

$$\sup\{|\,f_n(x)-f_m(x)\,|:\,x\in A\}\leq|\,a_n-a_m\,|,\; n,\; m\in\mathbb{N}$$

Prove that $\{f_n\}$ converges uniformly on A.

(b) If 5

$$f_n(x)=\frac{1}{2n^2}\log(1+n^4x^2)\;,\quad x\in[0,\,1],\; n\in\mathbb{N},$$

then prove that $\{f_n'(x)\}$ converges pointwise but not uniformly to $f'(x)$ on $[0, 1]$, where f is the uniform limit function of $\{f_n\}$.

6. (a) Let $f_n:D\longrightarrow\mathbb{R}$ be a continuous function on D, for $n\in\mathbb{N}$. If the series $\sum_{n=1}^{\infty} f_n$ be uniformly convergent on D, then prove that the sum function S is continuous on D. 4

(b) Study the continuity on $[0,\infty)$ of the function f defined by 4

$$f(x)=\sum_{n=1}^{\infty}\frac{x}{((n-1)\,x+1)(nx+1)}$$

7. (a) Let the series $\sum\limits_{n=1}^{\infty} f_n(x)$, $x \in A$, converges uniformly on A and that $f : A \longrightarrow \mathbb{R}$ be bounded. Prove that the series $\sum\limits_{n=1}^{\infty} f(x)\, f_n(x)$ converges uniformly on A. 3

(b) Let the series $\sum\limits_{n=1}^{\infty} f_n(x)$ of continuous functions on $[a, b]$ converge uniformly on $[a, b]$ and $g(x)$ be bounded and integrable on $[a, b]$. Prove that 5

$$\int_{\alpha}^{\beta} f(x)g(x)\,dx = \sum_{n=1}^{\infty} \int_{\alpha}^{\beta} f_n(x)g(x)\,dx,$$

where $a \le \alpha < \beta \le b$, and the convergence of the series of integrals is uniform on $[a, b]$.

8. (a) Let R be the radius of convergence of the power series $\sum\limits_{n=0}^{\infty} a_n x^n$, where $0 < R < \infty$. Prove that the series converges uniformly on $[-r, r]$ for any $0 < r < R$. 4

(b) Let the radius of convergence of $\sum\limits_{n=0}^{\infty} a_n x^n$ be r. Find the radius of convergence of $\sum\limits_{n=0}^{\infty} a_n x^{2n}$. 4

9. (a) Let $f(x) = \begin{cases} \dfrac{\pi}{2} + x \ , & -\pi \le x \le 0 \\ \dfrac{\pi}{2} - x \ , & 0 \le x \le \pi. \end{cases}$ 5

Show that $f(x) = \dfrac{4}{\pi} \sum\limits_{n=1}^{\infty} \dfrac{\cos(2n-1)x}{(2n-1)^2}$, where $x \in [-\pi, \pi]$.

(b) Examine whether the series $\sum\limits_{n=1}^{\infty} \dfrac{\sin(nx)}{\sqrt{n}}$ is a Fourier Series. 3

10.(a) Examine the convergence of the integrals $\int_{2}^{\infty} \dfrac{x^2}{\sqrt{x^7+1}}\,dx$ and $\int_{2}^{\infty} \dfrac{x^3}{\sqrt{x^7+1}}\,dx$. 4

(b) Show the convergence of $\int_{0}^{\infty} \left(\dfrac{x}{x+1} \right) \sin(x^2)\,dx$. 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2021

MTMACOR14T-MATHEMATICS (CC14)

RING THEORY AND LINEAR ALGEBRA II

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Either prove or disprove : If F is a field, then $F[x]$ is a field.

(b) Show that in an integral domain D, any two gcd's of two elements, if they exists are associates.

(c) Find all associates of $1+i$ in $\mathbb{Z}[i]$.

(d) If α, β be any two vectors in a Euclidean space V, then prove that

$$\|\alpha+\beta\| \leq \|\alpha\| + \|\beta\|$$

(e) Let V be an inner product space over a field $F(\mathbb{R}$ or $\mathbb{C})$ and $y, z \in V$. If $\langle x, y\rangle = \langle x, z\rangle,\ \forall x \in V$, then show that $y = z$.

(f) In Euclidean space $\mathbb{R}^3$ with standard inner product, let P be the subspace generated by the vectors (1, 1, 0) and (0, 1, 1). Find $P^{\perp}$.

(g) Find the minimum polynomial of the matrix

$$\begin{pmatrix} 3 & -1 & 0 \\ 0 & 2 & 0 \\ 1 & -1 & 2 \end{pmatrix}$$

(h) Let T be the linear operator on $\mathbb{R}^2$ defined by $T(a, b) = (2a+5b, 6a+b)$ and β be the standard ordered basis for $\mathbb{R}^2$. Find the characteristic polynomial of T.

(i) Let V be a finite dimensional inner product space and let T and U be linear operators on V. Show that $(TU)^* = U^*T^*$.

2. (a) Define a polynomial ring. 2+3

If D is an integral domain, show that $D[x]$ is an integral domain.

(b) Let $f(x)=x^4+[3]x^3+[2]x^2+[2]$, $g(x)=x^2+[2]x+[1]\in\mathbb{Z}_5[x]$. 3

Find $q(x), r(x)\in\mathbb{Z}_5[x]$ such that $f(x)=q(x)g(x)+r(x)$, where either $r(x)=0$ or, $0\le\deg r(x)<\deg g(x)$.

3. (a) Show that in an integral domain D, every prime element is irreducible. 3

(b) Consider the integral domain $\mathbb{Z}[i\sqrt{5}]=\{a+bi\sqrt{5}: a, b\in\mathbb{Z}\}$. Show that $3=3+0.i\sqrt{5}\in\mathbb{Z}[i\sqrt{5}]$ is irreducible but not a prime. 3

(c) Test for the irreducibility of the polynomial $x^3+[3]x+[4]$ over $\mathbb{Z}_5$. 2

4. (a) Let R be a commutative ring with 1. If $R[x]$ is a principal ideal domain, show that R is a field. 4

(b) Show that $\mathbb{Z}[x]$ is not a principal ideal domain. 1

(c) Show that in a unique factorization domain, every irreducible element is prime. 3

5. (a) Prove that the eigenvalues of a real symmetric matrix are all real. 3

(b) Find the eigenvalues and the corresponding eigen vectors of the following real matrix: 5

$$\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

6. (a) Prove that every square matrix satisfies its own characteristic equation. 4

(b) Diagonalize the following matrix orthogonally: 4

$$\begin{pmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{pmatrix}$$

7. (a) Show that an orthogonal set of non-null vectors in an Euclidean space V is linearly independent. 3

(b) Apply the Gram-Schmidt process to the vectors $\beta_1=(1,0,1)$, $\beta_2=(1,0,-1)$, $\beta_3=(0,3,4)$ to obtain an orthonormal basis for $\mathbb{R}^3$ with the standard inner product. 5

8. (a) In an inner product space V prove that $|\langle\alpha,\beta\rangle|\le\|\alpha\|\,\|\beta\|$, for all $\alpha,\beta\in V$. 4

(b) Let V be a finite dimensional inner product space and f be a linear functional on V. Then show that there exists a unique vector β in V such that $f(\alpha)=\langle\alpha,\beta\rangle$, for all $\alpha\in V$. 4

9. (a) Let V and W be vector spaces over the same field F of dimension n and m respectively. Prove that the space $L(V, W)$ has dimension mn. 4

(b) The matrix of $T:\mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is given by 2+2

$A=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ relative to the standard ordered basis of $\mathbb{R}^2$.

Find T and T^*, where T^* is the adjoint of T.

10.(a) Let T be a linear operator on a vector space V. Define a T-invariant subspace of V. 2+1+1

Let T be the linear operator on $\mathbb{R}^3$ defined by

$$T(a, b, c)=(a+b, b+c, 0).$$

Show that the xy-plane $=\{(x, y, 0): x, y \in \mathbb{R}\}$ and the x-axis $=\{(x, 0, 0): x \in \mathbb{R}\}$ are T-invariant subspaces of $\mathbb{R}^3$.

(b) Find the dual basis of the basis $\beta=\{(2,1),(3,1)\}$ of $\mathbb{R}^2$. 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2021

MTMADSE04T-MATHEMATICS (DSE3/4)

THEORY OF EQUATIONS

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) If x^4+px^2+qx+r can be expressed in the form $(x-a)^3(x-b)$, show that $8p^3+27q^2=0$.

(b) Find the quotient and remainder when x^3+5x^2+1 is divided by $x+3$.

(c) If $\alpha,\ \beta,\ \gamma$ be the roots of $x^3+qx+r=0$, prove that $\sum\frac{1}{\beta+\gamma-\alpha}=\frac{q}{2r}$.

(d) The equation $x^n-nx+n-1=0\,(n>1)$ is satisfied by $x=1$. What is the multiplicity of this root?

(e) Use Strum's theorem separate the roots of the equation $3x^4-6x^2-8x-3=0$.

(f) Verify whether the following functions are symmetric or not:

(i) $f(x,\ y,\ z)=x^2y^2+y^2z^2+z^2x^2$

(ii) $f(x,\ y,\ z)=xy+yz$.

(g) Multiply the roots of the equation $x^4+\frac{1}{2}x^3+\frac{1}{4}x+\frac{5}{12}=0$ by a suitable constant so that the fractional co-efficients of the equation may be removed.

(h) Define reciprocal equation.

(i) Find the number of special roots of the equation $x^n-1=0$, when n is a prime and $n=p^\alpha$, where p is a prime and α is a positive integer > 1.

2. (a) If $f(x)$ be a polynomial then prove that $(x-\alpha)$ is a factor of $f(x)$ if and only if $f(\alpha)=0$. 2+2

(b) Show that $x^{20}+x^{15}+x^{10}+x^5$ is divisible by x^2+1. 4

3. (a) If α be a special root of the equation $x^n-1=0$, then prove that $\frac{1}{\alpha}$ is also a special root of it. 4

(b) If α be a special root of the equation $x^{12}-1=0$, prove that $(\alpha+\alpha^{11})(\alpha^5+\alpha^7)=-3$. 4

4. (a) Find the equation whose roots are the roots of the equation $x^3+3x^2-8x+1=0$ (i) each diminished by 4, (ii) increased by 1. 2+2

(b) Find the relation among the coefficients of the equation $a_0x^3+3a_1x^2+3a_2x+a_3=0$ so that the second term and the third term may be removed by the transformation $x=y+h$. 4

5. (a) Find an upper limit of the real roots of the equation $x^4-2x^3+3x^2-2x+2=0$. 4

(b) Calculate Sturm's functions and locate the position of the real root of the equation $x^4-x^2-2x-5=0$. 4

6. (a) If an equation with rational coefficients has a surd root of $\alpha+\sqrt{\beta}$, where α, β are rational and β is not a perfect square, then show that it has the conjugate root $\alpha-\sqrt{\beta}$. 4

(b) Determine r so that one root of the equation $x^3-rx^2+rx-4=0$ shall be reciprocal of another and find all the roots. 4

7. (a) State Newton's theorem. If $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ be the roots of the equation $x^4+p_2x^2+p_3x+p_4=0$. Find the value of $\sum\alpha^3$ by Newton's theorem. 2+2

(b) Solve $3x^3-26x^2+52x-24=0$ given that the roots are in geometric progression. 4

8. (a) Reduce the biquadratic $2x^4-4x^3+3x^2+2x+3=0$ into standard form. 4

(b) If α, β, γ be the roots of $x^3+px+q=0$, prove that $6S_5=5S_2S_3$, where $S_r=\sum\alpha^r$. 4

9. (a) Find the number and position of the real roots of the equation $x^5-5x+1=0$. 6

(b) Show that the equation $(x-a)^3+(x-b)^3+(x-c)^3+(x-d)^3=0$, where a, b, c, d are not all equal, has only one real root. 2

10.(a) If α, β, γ be the roots of the equation $x^3+px^2+qx+r=0$, find an equation whose roots are $\frac{1}{\alpha}+\frac{1}{\beta}-\frac{1}{\gamma}$, $\frac{1}{\beta}+\frac{1}{\gamma}-\frac{1}{\alpha}$, $\frac{1}{\alpha}+\frac{1}{\gamma}-\frac{1}{\beta}$. 6

(b) If $\alpha, \beta, \gamma, \delta$ be the roots of $x^4-3x^3+4x^2-5x+6=0$, find the value of $(\alpha^2+3)(\beta^2+3)(\gamma^2+3)(\delta^2+3)$. 2

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——

लक्ष्यं सिद्धमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2021

MTMADSE05T-MATHEMATICS (DSE3/4)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) What is an isotone from a poset P into a poset Q? Show that each of the operations of join and meet in a lattice L induces an isotone from L into itself.

(b) Define direct product of two ordered sets P and Q. Prove that the direct product LM of two lattices L and M is a lattice.

(c) Let L be a distributive lattice and $c, x, y \in L$. If in L, $c \wedge x = c \wedge y$ and $c \vee x = c \vee y$, then show that $x = y$.

(d) Karnaugh map for a Boolean polynomial $f(x, y, z)$ in three Boolean variables x, y, z is given below:

$z \backslash xy$	00	01	11	10
0	0	1	1	1
1	0	0	0	1

Determine $f(x, y, z)$ from the above map and then express it in DNF in the variables x, y, z.

(e) Simplify the regular expression

$$01^*1 + 11^*01^*1 + (0 + 01^*1)1^*1$$

on the alphabet $\Sigma = \{0, 1\}$.

(f) State the Pumping Lemma for regular languages.

(g) Define a Turing machine.

(h) Construct a context-free grammar that generates the following language:

$$\{\omega c \omega^R : \omega \in \{0, 1\}^*\}$$

(i) Let $\Sigma = \{a, b\}$. Write regular expression for the following set:

All strings in Σ^* with exactly one occurrence of the substring *aaa*

2. (a) Consider the ordered set $(\mathbb{N}, \leq)$, where for any $a, b \in \mathbb{N}$, $a \leq b$ if and only if $a \mid b$. Show that $(\mathbb{N}, \leq)$ is a lattice. Is this lattice complete? Justify your answer. 2

(b) Let P be a finite ordered set. Then, for any $x, y \in P$, prove that $x < y$ if and only if there exists a finite sequence of covering relation $x = x_0 \prec x_1 \prec \ldots \prec x_n = y$. 2

(c) Let P and Q be two finite ordered sets and $\phi: P \to Q$ be a bijective mapping. Then, prove that the following assertions are equivalent: 4

(i) ϕ is an order isomorphism

(ii) $x \prec y$ in $P \Leftrightarrow \phi(x) \prec \phi(y)$ in Q.

3. (a) Let $f: B \to C$, where B and C are Boolean algebras. 5

(i) Assume f to be a lattice homomorphism. Then prove the following to be equivalent:

- $f(0) = 0$ and $f(1) = 1$
- $f(a') = (f(a))', \ \forall a \in B$.

(ii) Also prove that, if f preserves$'$, then (f preserves $\vee$ if and only if f preserves $\wedge$).

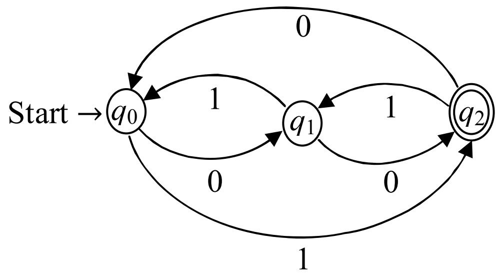
(b) Let L and K be lattices and $f: L \to K$ a map. Prove the following to be equivalent: 3

(i) f is order preserving

(ii) $(\forall a, b \in L)\ f(a \vee b) \geq f(a) \vee f(b)$

(iii) $(\forall a, b \in L)\ f(a \wedge b) \leq f(a) \wedge f(b)$.

4. (a) Convert the following DFA to a regular expression. 4



(b) Let $\Sigma = \{a, b\}$. Let L be a language over Σ, consisting of strings of length at least 2, where the first letter is the same as the last letter, and the second letter is the same as the second to last letter. For example, $a \notin L$, $b \notin L$, $aa \in L$, $aaa \in L$, $aba \in L$, $bbaabba \notin L$. Design a DFA that accepts L. 4

5. (a) Kleene closure of a regular language A, is defined as $A^* = \{x_1 x_2 \cdots x_k \mid k \geq 0$ and each $x_i \in A\}$. Prove that Kleene closure of every regular language is regular. 4

(b) Prove that every non-deterministic finite automaton can be converted to an equivalent one that has a single accept state. 4

6. (a) Draw the state diagram of a Pushdown Automata realizing $\{0^n 1^n \mid n \geq 0\}$. 3

(b) Prove that any context-free language is generated by a context-free grammar in Chomsky normal form. 5

7. (a) Prove that the normal subgroups of a group form a modular lattice, under set inclusion. 3

(b) How many minimal Boolean polynomials are there, in n Boolean variables? When is a Boolean polynomial said to be in Disjunctive Normal Form (DNF)? 2

(c) Convert the Boolean polynomial 3

$$f(x, y, z, t) = xyzt + x'y'zt + xyz't + xy'z't + xyz't' + xy'z't'$$

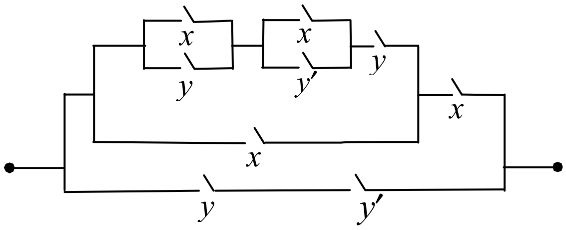
into its minimal form.

8. (a) Truth table for a Boolean polynomial $f(x, y, z)$ in three variables x, y, z is given below: 5

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

From this table, form Karnaugh map for the polynomial $f(x, y, z)$, taking x along the row and yz along the column of the map. From the Karnaugh map determine $f(x, y, z)$. Then, construct the logic circuit (of the logic-gates) for $f(x, y, z)$.

(b) Find the Boolean polynomial which represents the following switching circuit: 2+1



Hence, draw an equivalent circuit as simple as possible.

9. (a) Let $G = (V, \Sigma, S, P)$ be a context-free grammar (CFG), where $V = \{S, A, B\}$ is the set of non-terminals, $\Sigma = \{a, b, c\}$ is the set of terminals, S is the start symbol and P is the set of productions $S \to ABa,\ A \to aab,\ B \to Ac$. Transform this grammar G into a CFG in Chomsky Normal Form. 6

(b) Define ID of a Non-deterministic Pushdown Automaton (NPDA). 2

10.(a) Construct the NPDA which accepts the context-free language L on the alphabet $\Sigma = \{a, b, c\}$, generated by the CFG, G with variables A, B, C; the start variable S and productions 4

$$S \to aA$$
$$A \to aABC|bB|a$$
$$B \to b$$
$$C \to c$$

(b) Describe how can a Turing Machine be made as a unary to binary converter. 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2021

MTMADSE06T-MATHEMATICS (DSE3/4)

MECHANICS

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Define astatic equilibrium and astatic centre.

(b) Define coefficient of friction and the angle of friction.

(c) State the principle of virtual work for a system of coplanar forces acting on a rigid body.

(d) Find the centre of gravity of a uniform rectangular lamina with sides of length a and b.

(e) Weights proportional to 1, 4, 9 and 16 are placed in a straight line so that the distance between them are equal; find the position of their centre of gravity.

(f) Describe stable and unstable equilibrium and the position of the centre of gravity in each case.

(g) Write down the equations of motion of a particle projected with a velocity u making an angle α with the horizon in a medium offering resistance proportional to the velocity.

(h) Define equimomental bodies. State the necessary and sufficient condition for two systems to be equimomental.

(i) If a rigid body rotates about a space-fixed axis, θ be the angular velocity of the body about the axis at any instant and Mk^2 the moment of inertia of the body about the axis, then prove that the kinetic energy of the body at that instant is $\frac{1}{2}Mk^2\dot{\theta}^2$.

2. (a) Forces P, Q, R act along the x-axis, y-axis and the straight line $x\cos\alpha + y\sin\alpha = p$. Find the magnitude of the resultant and the equation of the line of action. 4

(b) A solid homogeneous hemisphere rests on a rough horizontal plane whose coefficient of friction is μ' and against a rough vertical wall with coefficient of friction is μ. Show that the least angle that the base of the hemisphere can make with the vertical is $\cos^{-1}\left(\frac{8\mu'}{3}\frac{1+\mu}{1+\mu\mu'}\right)$. 4

3. (a) Three forces P, Q, R act along the three straight lines $x=0$, $y-z=a$; $y=0$, $z-x=a$; $z=0$, $x-y=a$ respectively. Show that P, Q, R cannot reduce to a couple. 4

(b) The density at any point of a circular lamina varies as the n-th power of the distance from a point O on the circumference. Show that the centre of gravity of the lamina divides the diameter through O in the ratio $n+2:2$. 4

4. (a) State the principle of virtual work for any system of coplanar forces acting on a rigid body. 2

(b) A regular pentagon $ABCDE$ is formed of five uniform heavy rods, each of weight W and freely joined at their extremities. It is freely suspended from A and is maintained in its regular pentagon form by light rod joining B and E. prove that the stress in this rod is $W\cot(18°)$. 6

5. A solid hemisphere rests on a plane inclined to the horizon at an angle $\alpha < \sin^{-1}\frac{3}{8}$ and the plane is rough enough to prevent any sliding. Find the position of equilibrium and show that it is stable. 8

6. (a) If the axes Ox, Oy revolve with constant angular velocity w and the components of velocities of the point (x, y) are px and py, where $p=\frac{a^2-b^2}{a^2+b^2}w$, prove that the point describes relatively to the axes an ellipse. Find also its periodic time. 3+1

(b) A body describing an ellipse of eccentricity e under the action of a force directed to focus when at the nearer apse, the centre of force is transferred to the other focus. Prove that eccentricity of the new orbit is $e\frac{(3+e)}{(1-e)}$. 4

7. A particle is projected at right angles to the line joining it to a centre of force, attracting according to the law of inverse square of the distance, with a velocity $\frac{\sqrt{3}V}{2}$, where V is the velocity from infinity. Find the eccentricity of the orbit described and show that the periodic time is $2\pi T$, T being the time taken to describe the major axis of the orbit with velocity V. 8

8. A particle of given mass be moving in a medium whose resistance varies as the velocity of the particle. Show that the equation of the trajectory can, by a proper choice of axes be put into the form $y+ax=b\log x$. 8

9. (a) Using the necessary condition for a given straight line to be a principal axis at some point of its length, prove the following:

(i) through each point of a plane lamina there exists a pair of principal axis of the lamina, 2

(ii) if an axis passes through the centre of gravity of a body and is a principal axis at any point of its length, then it is a principal axis at all points of its length. 2

(b) Show that for a rigid body the motion of centre of inertia is independent of the motion relative to the centre of inertia. 4

10.(a) State the principle of conservation of momentum both for finite and impulsive forces. State also the principle of conservation of energy. 4

(b) A solid homogeneous cone of height h and vertical angle 2α, oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{h}{5}(4+\tan^2\alpha)$. 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-III Examination, 2021

MATHEMATICS

PAPER: MTMA-V

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Marks-28

Answer Question No. 1 and any *two* from the rest

1. Answer any ***two*** questions from the following: $3\times2=6$

(a) Show that the family of open intervals $I_n=\left\{x\in\mathbb{R}:\frac{1}{2^n}<x<2\right\}$, $n\in\mathbb{N}$ forms an open cover of (0, 1). Also show that this cover does not admit any finite subcover for (0, 1).

(b) Show that $\int\limits_1^\infty \frac{\cos x}{\sqrt{1+x^3}}\,dx$ converges absolutely.

(c) Examine if the sequence of functions $\{f_n\}$, $n\in\mathbb{N}$ defined by $f_n(x)=\frac{1-x^n}{1-x}$, $0<x<1$ converges uniformly on (0, 1).

(d) If $\sum\limits_{n=1}^{\infty}a_n$ converges absolutely then prove that $\sum\limits_{n=1}^{\infty}\frac{a_nx^n}{1+x^{2n}}$ converges uniformly on $\mathbb{R}$.

(e) Find the radius of convergence of the power series

$$1+2x+\frac{x^2}{2^2}+2^3x^3+\frac{x^4}{2^4}+\,.......$$

(f) Evaluate $\iint\limits_R xy^2\,dxdy$ where R is the region bounded by $x^2+y^2=1$.

(g) Let $f:[-2,\,2]\longrightarrow\mathbb{R}$ be defined by $f(x)=\frac{1}{3}x^3-x$. Show that f is a function of bounded variation on [–2, 2]. Also find $V_f[-2,\,2]$.

(h) If e be defined by $\int\limits_1^e\frac{dt}{t}=1$, prove that $2<e<3$.

(i) Let $f:[0,\,3]\longrightarrow\mathbb{R}$ be defined by $f(x)=[x]$. Show that f is Riemann integrable on [0, 3] and compute $\int\limits_0^3[x]\,dx$.

2. (a) Define compact set. Find which of the following sets is compact: 1+2

   (i) $A=\{\frac{1}{n}:\ n\in\mathbb{R}\}\cup\{0\}$, (ii) $\{x\in\mathbb{Q}:\ 0\le x\le 1\}$.

   (b) Prove that every sequence in a compact set C has a convergent subsequence with limit in C. 4

   (c) Prove that if a function $f:C\longrightarrow\mathbb{R}$, where $C\subset\mathbb{R}$ is compact, is continuous then it is bounded and attains its bounds. 4

3. (a) Let $f_n:[a,b]\longrightarrow\mathbb{R}$ be Riemann integrable for all $n\in\mathbb{N}$ and let the sequence $\{f_n\}$ be uniformly convergent to f on $[a,b]$. Show that f is Riemann integrable on $[a,b]$ and $\int_a^b f(x)\,dx=\lim\limits_{n\to\infty}\int_a^b f_n(x)\,dx$. 4

   (b) The sequence of functions $\{f_n\}$ is defined by $f_n(x)=\dfrac{nx}{1+nx}$, $x\ge 0$, $n\in\mathbb{N}$. Find f so that $\lim\limits_{n\to\infty}f_n(x)=f(x)$ for all $x\ge 0$. Show that for any $a>0$, $\{f_n\}$ converges to f uniformly on $[a,\infty)$ but converges pointwise on $[0,\infty)$. 1+3

   (c) For each $n\in\mathbb{N}$ let S_n be the set defined by $S_n=\{\frac{p}{q}:\ p,q\in\mathbb{N},\ 1\le p\le q\le n;$ p, q are prime to each other$\}$. For each $n\in\mathbb{N}$ define $f_n:[0,1]\longrightarrow\mathbb{R}$ by 1+2

$$\begin{aligned} f_n(x) &= 0\ ,\quad x\in S_n\\ &= 1\ ,\quad x\in[0,1]-S_n \end{aligned}$$

   Find pointwise limit of the sequence $\{f_n\}$. Does the sequence $\{f_n\}$ converge uniformly? Justify your answer.

4. (a) State Abel's test for uniform convergence of a series. Use it to show that the series $\sum\limits_{n=1}^{\infty}\dfrac{e^{-nx}}{2^n}$ is uniformly convergent in $[a,\infty)$ for any $a\in\mathbb{R}$. 1+3

   (b) Express $f(x)=\dfrac{1}{1-x}$ as a power series. Hence prove that $\log(1-x)=-\sum\limits_{n=1}^{\infty}\dfrac{x^{n+1}}{n+1}$, give proper justification. 1+3

   (c) Find the radius of convergence of the power series $\sum\limits_{n=1}^{\infty}\dfrac{(n\,!)^2}{(2n)!}x^n$. 3

5. (a) Show that a function continuous on a closed interval is Riemann-integrable there. 4

   (b) Verify whether the function $f:[0,1]\longrightarrow\mathbb{R}$, defined by 3

$$\begin{aligned} f(x) &= 1+\frac{(-1)^n}{2^n}\quad \text{when}\quad \frac{1}{n+1}<x\le\frac{1}{n}\ ,\ n\in\mathbb{N}\\ &= 0\qquad \text{when}\quad x=0, \end{aligned}$$

   is Riemann-integrable.

(c) Assume that $f:[a,b]\to \mathbb{R}$ is a Riemann-integrable function and the function $F:[a,b]\to \mathbb{R}$ is continuous on $[a,b]$ and differentiable on (a,b) such that $F'(x)=f(x)$ for all $x\in(a,b)$. Show that $\int_a^b f(x)\,dx = F(b)-F(a)$. 4

6. (a) Find the values of p and q for which the integral $\int_a^b \frac{dx}{(x-a)^p(b-x)^q}$ is convergent. 6

(b) Define gamma function. Show that $\Gamma(n+1)=n\Gamma(n)$, $n>0$. Hence find $\Gamma(n)$ when n is a positive integer. 1+2+2

7. (a) Define $\ln(x)$ as integral function. Show that for $x, y>0$, $\ln(xy)=\ln(x)+\ln(y)$. 1+2

(b) Using Second Mean Value Theorem of Integral Calculus prove that 3

$$\left|\int_a^b \frac{\cos x}{1+x}\,dx\right| < \frac{4}{1+a}, \quad 0<a<b.$$

(c) Let $f:[a,b]\times[c,d]\longrightarrow \mathbb{R}$ be a continuous function. Define $\phi:[c,d]\longrightarrow \mathbb{R}$ by $\phi(y)=\int_a^b f(x,y)\,dx$, $c\le y\le d$. Show that ϕ is continuous on $[c,d]$. State a sufficient condition for differentiability of ϕ. 4+1

8. (a) Define (i) a function of bounded variation (ii) total variation of a function of bounded variation. 2+1

(b) Show that a function of bounded variation can be expressed as a difference of two monotonically increasing functions. 4

(c) Find the length of the cardioid given by $r=a(1+\cos\theta)$, $0\le\theta\le\pi$. 4

9. (a) Test the continuity of the function 3

$$\begin{aligned} f(x,y) &= \frac{x^3y}{x^6+y^2}, \quad (x,y)\neq(0,0) \\ &= 0, \quad (x,y)=(0,0) \end{aligned}$$

at the point (0, 0).

(b) Expand $f(x,y)=e^{ax}\cos by$ by Taylor's Theorem about (0, 0) up to the terms of degree two. 4

(c) Using Lagrange's method of multiplier find the points on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ which are farthest and nearest points to the point (5, 1). 4

10.(a) Obtain the Fourier series for f where $f:[-\pi,\pi]\longrightarrow\mathbb{R}$ is defined by 3

$$f(x)=\begin{cases} \cos x, & 0\le x\le\pi \\ -\cos x, & -\pi\le x<0 \end{cases}$$

(b) Find the value of 4

$$\iiint_E \frac{1}{(1+x+y+z)^3}\,dx\,dy\,dz$$

where E is the tetrahedron bounded by the planes $x=0,\ y=0,\ z=0$, $x+y+z=1$.

(c) Show by changing the order of integration that 4

$$\int_0^1 dx \int_x^{1/x} \frac{y}{(1+xy)^2(1+y^2)}\,dy = \frac{\pi-1}{4}$$

GROUP-B

(Marks-11)

Answer any *one* question from the following $11\times1=11$

11.(a) If $C[0,\,1]$ be the set of all real-valued continuous functions on $[0,\ 1]$ and for $f,\,g \in C[0,\,1]$, $d: C[0,\,1]\times C[0,\,1] \longrightarrow \mathbb{R}$ be defined by 4

$$d(f,\,g) = \sup_{x\in[0,\,1]} |\,f(x)-g(x)\,|,$$

then establishing the validity of $d(f,\,g)$, show that $C[0,\,1]$ forms a metric space with respect to the metric *d*.

(b) In a metric space $(X,\,d)$, prove that a set F is closed (i.e. $X\setminus F$ is open) if and only if $F^d \subseteq F$, F^d is the derived set of F. 2

(c) Show that a Cauchy sequence in a metric space $(X,\,d)$ is bounded. 2

(d) Let $(Q,\,d)$ be the metric space of rational numbers with usual metric. Show that $\{x_n\}$, where $x_n = 1+\frac{1}{1\,!}+\frac{1}{2\,!}+\,......\,+\frac{1}{(n-1)\,!}$, $n\in\mathbb{N}$ is a Cauchy sequence in $(Q,\,d)$ and hence show that $(Q,\,d)$ is not a complete metric space. 3

12.(a) Let $(X,\,d)$ be a metric space and $d^*: X\times X \to \mathbb{R}$ be defined by 3+1

$$d^*(x,\,y) = \frac{d(x,\,y)}{1+d(x,\,y)},\quad x,\,y\in X$$

Prove that $(X,\,d^*)$ is a metric space. Also show that the metric d^* is bounded.

(b) Let $(X,\,d)$ be a metric space and A be a subset of X. If $d(x,\,A)=0$, for $x\in X$ then prove that $x\in \overline{A}$, where $\overline{A}$ is the closure of A. 2

(c) If $\{x_n\}$, $\{y_n\}$ be two sequences in a metric space $(X,\,d)$ such that $d(x_n,\,y_n)\to 0$ and $\{x_n\}$ be a Cauchy sequence then prove that $\{y_n\}$ is a Cauchy sequence in $(X,\,d)$. 2

(d) In a metric space $(X,\,d)$, let $\{F_n\}$ be a nested sequence of non-empty closed sets in X such that diam $F_n \to 0$ as $n\to\infty$. Prove that $F=\bigcap_{n=1}^{\infty} F_n$ consists of exactly one point. 3

GROUP-C

(Marks-11)

Answer any *one* question from the following $11\times1 = 11$

13.(a) Consider the sphere $S=\{(x_1, x_2, x_3)\in \mathbb{R}^3 : x_1^2+x_2^2+x_3^2=1\}$ in $\mathbb{R}^3$. Let $n=(0, 0, 1)\in S$. Suppose that $\mathbb{C}$ is identified with the plane $P: x_3=0$ in $\mathbb{R}^3$ by $\omega = x+iy \equiv (x, y, 0)$. Show that the spherical representation of any $\omega\in\mathbb{C}$ is given by $x_1=\dfrac{\omega+\overline{\omega}}{|\omega|^2+1}$, $x_2=\dfrac{-i(\omega-\overline{\omega})}{|\omega|^2+1}$, $x_3=\dfrac{|\omega|^2-1}{|\omega|^2+1}$. 4

(b) Let $f: D \longrightarrow \mathbb{C}$ be a function defined on a subset D of $\mathbb{C}$. Let z_0 be a limit point of D. Show that $\lim\limits_{z\to z_0} f(z)=\omega$ if and only if $\lim\limits_{n\to\infty} f(z_n)=\omega$ for any sequence $\{z_n\}$ in D such that $\lim\limits_{n\to\infty} z_n = z_0$. 5

(c) Prove that the function $f(z)=\overline{z}$ is nowhere differentiable on $\mathbb{C}$. 2

14.(a) Let $f: G \longrightarrow \mathbb{C}$ be a function defined on an open subset G of $\mathbb{C}$, where 5

$$f(x+iy)=u(x, y)+iv(x, y)$$

Let f be differentiable at a point $z_0=x_0+iy_0\in G$. Show that $u(x, y)$ and $v(x, y)$ are differentiable at (x_0, y_0) and Cauchy-Riemann equations are satisfied at (x_0, y_0). Further show that

$$f'(z_0) = \frac{\partial u}{\partial x}-i\frac{\partial u}{\partial y} = \frac{\partial v}{\partial y}+i\frac{\partial v}{\partial x}$$

(b) Show that the function $f(z)=\begin{cases}\dfrac{(\overline{z})^2}{z} & ; \ z\neq 0\\ 0 & ; \ z=0\end{cases}$ 4

is not differentiable at the origin even though it satisfies Cauchy-Riemann equations there.

(c) Show that $u(x, y)=x^3-3xy^2$ is a harmonic function. Determine a conjugate harmonic function of u. 2

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2021

MTMHGEC02T/MTMGCOR02T-MATHEMATICS (GE2/DSC2)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Test whether the equation $xdx + ydy + \dfrac{xdy - ydx}{x^2 + y^2} = 0$ is exact or not.

(b) Find an integrating factor of the differential equation $(x \log x)\dfrac{dy}{dx} + y = 2\log x$.

(c) Find particular integral of the differential equation $2x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} = \dfrac{1}{x}$.

(d) Find the transformation of the differential equation $x^2\dfrac{d^2y}{dx^2} - 5y = \log x$, using the substitution $x = e^z$.

(e) Find complementary function of the differential equation $x^2\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} = 3x$.

(f) Find the Wronskian of $y_1(x) = e^{-2x}$, $y_2(x) = xe^{-2x}$.

(g) Construct a PDE by eliminating a and b from $z = ae^{-b^2t}\cos bx$.

(h) Determine the order, degree and linearity of the following PDE:

$$\frac{\partial z}{\partial x} = \left(\frac{\partial^2 z}{\partial x^2}\right)^{5/2} + \left(\frac{\partial^2 z}{\partial y^2}\right)^{5/2}$$

(i) Classify the following PDE

$$(1+x^2)\, z_{xx} + (1+y^2)\, z_{yy} + xz_x + yz_y = 0$$

into elliptic, parabolic and hyperbolic for different values of x and y.

2. (a) Find an integrating factor of the differential equation 4

$$(2xy^4e^y + 2xy^3 + y)\, dx + (x^2y^4e^y - x^2y^2 - 3x)\, dy = 0$$

and hence solve it.

(b) Solve: $x\cos x\dfrac{dy}{dx}+y\,(x\sin x+\cos x)=1$ 4

3. (a) Find the curve for which the area of the triangle formed by x-axis, a tangent and the radius vector of the point of tangency is constant and equal to a^2. 4

(b) Using the substitution $u=\dfrac{1}{x}$ and $v=\dfrac{1}{y}$, reduce the equation $y^2(y-px)=x^4p^2$ to Clairaut's form and hence solve it. Here $p\equiv\dfrac{dy}{dx}$. 4

4. (a) Show that each of the functions e^x, e^{4x} and $2e^x-3e^{4x}$ is solution of the differential equation $\dfrac{d^2y}{dx^2}-5\dfrac{dy}{dx}+4y=0,\ -\infty<x<\infty$. 2+1+1+1

Are the three independent? If not, find which two of these are independent. Write down a general solution of the equation.

(b) Find the value of h so that the equation $(ax+hy+g)\,dx+(3x+by+f)\,dy=0$ becomes an exact differential equation. 3

5. (a) Solve by the method of variation of parameters: 5

$$(D^2-3D+2)y=e^x(1+e^x)^{-1},\ \text{where}\ D\equiv\frac{d}{dx}$$

(b) Find particular integral of the differential equation 3

$$(D^2+5D+6)y=e^{-2x}\sin 2x,\quad \text{where}\ D\equiv\frac{d}{dx}$$

6. (a) Solve in the particular cases: 5

$$\frac{d^2x}{dt^2}-4\frac{dx}{dt}+5x=0\ \text{giving that}\ x=1\ \text{and}\ \frac{dx}{dt}=2\ \text{when}\ x=0$$

(b) Solve: $\dfrac{d^2y}{dx^2}=x^2\sin x$ 3

7. (a) Solve the following total differential equation: 4

$$yz\,dx+2zx\,dy-3xy\,dz=0$$

(b) Solve: $x^2\dfrac{d^2y}{dx^2}+3x\dfrac{dy}{dx}+y=x\log x$ 4

8. (a) Form a PDE by eliminating the arbitrary function ϕ from 4

$$lx+my+nz=\phi(x^2+y^2+z^2)$$

(b) Solve the partial differential equation by Lagrange's method $x^2p+y^2q=(x+y)z$. 4

9. (a) Find the partial differential equation of planes having equal intercepts along x axis and y axis. 4

(b) Find $f(y)$ such that the total differential equation $\left(\frac{yz+z}{x}\right)dx - zdy + f(y)\,dz = 0$ is integrable. 4

10.(a) Formulate a PDE from the relation $f\left(\frac{x-a}{z-c}, \frac{y-b}{z-c}\right) = 0$. 3

(b) Find the Wronskian of x and $|x|$ in $[-1, 1]$. 2

(c) Solve $x^2 \frac{d^2y}{dx^2} - 6y = 0$. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——