# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2021-22

## MTMACOR01T-MATHEMATICS (CC1)

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) If $y=\sin kx+\cos kx$, prove that $y_n = k^n\{1+(-1)^n \sin 2kx\}^{1/2}$.

(b) Find the asymptotes of the curve $x=\dfrac{t^2}{1+t^3}$ , $y=\dfrac{t^2+2}{t+1}$.

(c) Determine $a$ such that, $\lim\limits_{x\to 0}\dfrac{a\sin x-\sin 2x}{\tan^3 x}$ exists and = 1.

(d) Determine the angle of rotation of the axes so that the equation $x+y+2=0$ may reduce to the form $ax+b=0$.

(e) Find the centre and radius of the sphere $x^2+y^2+z^2-4x+6y-8z=71$.

(f) Find the values of $a$ for which the plane $x+y+z=a\sqrt{3}$ touches the sphere $x^2+y^2+z^2-2x-2y-2z-6=0$.

(g) Find the equation of the cylinder whose generating line is parallel to $z$-axis and the guiding curve is $x^2+y^2=z$ , $x+y+z=1$.

(h) Show that the differential equation $\left|\dfrac{dy}{dx}\right|+|y|=0$ has a particular solution which is bounded.

(i) Obtain the singular solution of the differential equation $y-px-\frac{1}{p}=0$, where $p=\dfrac{dy}{dx}$.

2. (a) If $P_n=D^n(x^n\log x)$ then prove that $P_n=nP_{n-1}+(n-1)\,!$. Hence prove that $P_n=n\,!(\log x+1+\frac{1}{2}+\frac{1}{3}+\ldots\ldots\ldots\ldots+\frac{1}{n})$. 4

(b) If $x^{2/3}+y^{2/3}=c^{2/3}$ is the envelope of the lines $\frac{x}{a}+\frac{y}{b}=1$ where $a$, $b$ are variable parameters and $c$ is a constant then prove that $a^2+b^2=c^2$. 4

3. (a) Prove that the length of the loop of the curve $x=t^2$, $y=t-\frac{t^3}{3}$ is $4\sqrt{3}$. 4

(b) Find the asymptotes of the curve $x^2(x+y)(x-y)^2+2x^3(x-y)-4y^3=0$. 4

4. (a) Find the range of values of $x$ for which the curve $y=x^4-16x^3+42x^2+12x+1$ is concave or convex with respect to the $x$-axis and identify the points of inflexion if any. 4

(b) If $y=\sin(m\sin^{-1}x)$, show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$. 4

5. (a) Find the equation of the generating lines of the hyperboloid $3xy+yz+2zx+6=0$ which passes through the point (–1, 0, 3). 4

(b) Reduce the equation $4x^2+4xy+y^2-4x-2y+a=0$ to the canonical form and determine the type of the conic for different values of $a$. 4

6. (a) Find the equation of the cone whose vertex is (1, 0, –1) and which passes through the circle $x^2+y^2+z^2=4$, $x+y+z=1$. 4

(b) Find the equation of the curve in which the plane $z=h$ cuts the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ and find the area enclosed by the curve. 4

7. (a) The section of the cone whose guiding curve is the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $z=0$ by the plane $x=0$ is a rectangular hyperbola. Show that the locus of the vertex is the surface $\frac{x^2}{a^2}+\frac{(y^2+z^2)}{b^2}=1$. 4

(b) Show that the equation of the circle, which passes through the focus of the parabola $\frac{2a}{r}=1+\cos\theta$ and touches it at a point $\theta=\alpha$, is given by $r\cos^3\frac{\alpha}{2}=a\cos(\theta-\frac{3}{2}\alpha)$. 4

8. (a) Show that the general solution of the equation $\frac{dy}{dx}+Py=Q$ can be written in the form $y=k(u-v)+v$ where $k$ is a constant, $u$ and $v$ are its two particular solutions. 4

(b) Determine the curve in which the area enclosed between the tangent and the coordinate axes is equal to $a^2$. 4

9. (a) Solve $y(xy+2x^2y^2)\,dx+x(xy-x^2y^2)\,dy=0$. 4

(b) Reduce the equation $\sin y\frac{dy}{dx}=\cos x(2\cos y-\sin^2 x)$ to a linear equation and hence solve it. 4

10.(a) Using the transformation $u=x^2$ and $v=y^2$ to solve the equation $xyp^2-(x^2+y^2-1)\,p+xy=0$, where $p=\frac{dy}{dx}$. 4

(b) Solve $(x^2y^3+2xy)\,dy=dx$. 4

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2021-22

## MTMACOR02T-MATHEMATICS (CC2)

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) If $a$, $b$, $c$ are all positive and $abc = k^3$, then prove that $(1+a)(1+b)(1+c) \geq (1+k)^3$.

(b) Solve the equation $3z^5 + 2 = 0$.

(c) Apply Descartes' rule of sign to determine the number of positive, negative and complex roots of the equation $x^5 - x^4 - 2x^2 + 2x + 1 = 0$.

(d) Prove that $2^{3n} - 1$ is divisible by 7 for all $n \in \mathbb{N}$.

(e) If $\gcd(a, b) = 1$, then show that $b \mid ap \Rightarrow b \mid p$.

(f) Find a map $f : \mathbb{N} \longrightarrow \mathbb{N}$ which is one to one but not onto.

(g) Let $f : A \to B$ be an onto mapping and $P$, $Q$ be subsets of $B$. Prove that $f^{-1}(P \cap Q) = f^{-1}(P) \cap f^{-1}(Q)$.

(h) Find the minimum number of non-real roots of the polynomial equation $x^8 + x^4 - x^2 = 0$.

(i) Give an example of a reflexive and symmetric relation on the set {1, 2, 3} which fails to be an equivalence relation on {1, 2, 3}.

2. (a) If $a_1, a_2, a_3, a_4$ be distinct positive real numbers and $s = a_1 + a_2 + a_3 + a_4$, then show that $\frac{s}{s-a_1} + \frac{s}{s-a_2} + \frac{s}{s-a_3} + \frac{s}{s-a_4} > 5\frac{1}{3}$. 3

(b) Show that $(n+1)^n > 2^n n!$. 2

(c) If $A$ be the area and $2s$ the sum of the three sides of a triangle, show that $A \leq \frac{s^2}{3\sqrt{3}}$. 3

3. (a) If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then prove that 4

$$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$$

(b) If $z_1, z_2$ are complex numbers such that $z_1+z_2$ and $z_1 \cdot z_2$ are both real then show that either $z_1, z_2$ are both real or $z_1=\bar{z}_2$. 4

4. (a) Solve the equation $x^3-3x-1=0$, by Cardan's method. 4

(b) Form a biquadratic equation with rational coefficients, two of whose roots are $\sqrt{3}\pm 2$. 4

5. (a) Let $X$ be any non-empty set. Prove that there does not exist any surjective map from $X$ to $P(X)$, the power set of $X$. 2

(b) Prove that the relation $\rho$ on $\mathbb{R}$ defined by $x\rho y$ if and only if $x-y\in\mathbb{Q}$ $(x,\ y\in\mathbb{R})$ is an equivalence relation. Find the equivalence class containing the element 0. 2+1

(c) A relation $\rho$ on $\mathbb{R}$ is defined as follows: 3

$$a\rho b \text{ if and only if } |a|\le b$$

Show that $\rho$ is transitive but neither reflexive nor symmetric.

6. (a) If $p$ is a prime greater than 3, then show that $2p+1$ and $4p+1$ can not be primes simultaneously. 2

(b) Use mathematical induction to prove that for any positive integer $n$ 3

$$1.2+2.2^2+3.2^3+\dots\dots+n.2^n=(n-1)2^{n+1}+2$$

(c) Prove that for any positive integer $n$, $3^{4n+2}+5^{2n+1}\equiv 0\ (\text{mod } 14)$. 3

7. Transform the matrix $A=\begin{pmatrix} 1 & 2 & -1 & 10 \\ -1 & 1 & 2 & 2 \\ 2 & 1 & -3 & 2 \end{pmatrix}$ to its row reduced echelon form. 4+2+2=8

Hence find $\operatorname{rank} A$ and the solution set of the system of linear equations given by

$$x+2y-z=10$$

$$-x+y+2z=2$$

$$2x+y-3z=2$$

8. (a) Use Cayley-Hamilton theorem to express $A^{-1}$ as a polynomial in $A$ and then compute $A^{-1}$ where $A=\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$. 2+2

(b) Show that the eigen values of an orthogonal matrix are of unit modulus. 4

9. (a) If $A$ be a square matrix, then show that the product of the characteristic roots of $A$ is $\det A$. 3

(b) Find all the eigen values of the following real matrix: 2+3

$$A=\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

Find one eigen vector corresponding to the largest eigen value found above.

10.(a) Express the matrix 3+2

$$A=\begin{pmatrix} 2 & 0 & 1 \\ 3 & 3 & 0 \\ 6 & 2 & 3 \end{pmatrix}$$

as product of elementary matrices and hence, find $A^{-1}$.

(b) If $A=\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -2 \\ 0 & 2 & 0 \end{pmatrix}$, show that $A^2$ cannot have imaginary characteristic roots. 3

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2022

## MTMACOR03T-MATHEMATICS (CC3)

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
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*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Using Archimedean property of $\mathbb{R}$, prove that the set of natural numbers, $\mathbb{N}$ is unbounded above.

(b) Find the supremum of the set $S=\left\{\frac{1}{p}+\frac{1}{q}\ :\ p,\ q\in\mathbb{N}\right\}$.

(c) For any two sets $S$ and $T$ in $\mathbb{R}$, prove that $\overline{S\cap T}\subseteq\overline{S}\cap\overline{T}$, where for any $A\subseteq\mathbb{R}$, $\overline{A}$ denotes the closure of $A$.

(d) If $A=\left[\frac{1}{3},\frac{8}{3}\right]$ and $B=\left(1,\frac{11}{3}\right)$, examine whether $A\cup B$ is compact or not.

(e) Find the set of all limit points of the set $E=\left\{\frac{n-1}{n+1}\ :\ n\in\mathbb{N}\right\}\cup\{2,\ 3\}$.

(f) Two sets $A$ and $B$ of real numbers are such that $A$ is closed and $B$ is compact. Prove that $A\cap B$ is compact.

(g) Show that $\left(\frac{n}{n+1}\right)_n$ is a Cauchy sequence.

(h) Apply Cauchy's root test to check the convergence of the series:

$$1+\frac{1}{2^3}+\frac{1}{2^2}+\frac{1}{2^5}+\frac{1}{2^4}+\ldots\ldots..$$

2. (a) Let $T$ be a bounded subset of $R$. If $S=\{|x-y|:\ x,\ y\in T\}$ then show that $\sup S=\sup T-\inf T$. 3

(b) Prove that the set of rational numbers is not order complete. 3

(c) If $A$ be an uncountable set and $B$ be a countable subset of $A$, then prove that $A-B$ is uncountable. 2

3. (a) Give example of a set which is 2+2
   (i) both open and closed,
   (ii) neither open nor closed.
   Give reasons in support of your answer.
   (b) Prove that every open interval is an open set and every open set is an union of open intervals. 2+2

4. (a) Let $H=(0,1)$ and $x\in H$. Let $\sigma=\{I_x :\ x\in H\}$, where $I_x=\left(\dfrac{x}{2},\dfrac{x+1}{2}\right)$. Show that $\sigma$ is an open cover of $H$ but it has no finite sub cover. 3
   (b) If $S$ and $T$ are compact sets in $R$ then show that $S\cup T$ is also compact. 2
   (c) Prove or disprove: Union of an infinite number of compact sets is compact. Give reasons in support of your answer. 3

5. (a) State Bolzano-Weierstrass theorem for the set of real numbers. Can you apply the theorem for the set of natural numbers? Justify your answer. 1+1
   (b) Show that the intersection of finite collection of open sets is an open set in $\mathbb{R}$. Give an example to show that arbitrary intersection of open sets may not be an open set. 2+1
   (c) Show that the set $A=\{x\in\mathbb{R} :\ \cos x\neq 0\}$ is an open set, but not a closed set. 2+1

6. (a) Show that the sequence $\left\{\dfrac{3^{2n}}{4^{3n}}\right\}$ is a null sequence. 2
   (b) Use Sandwich theorem to prove the following limit: 3
   $$\lim_{n\to\infty}\left[\frac{1}{n^2+1}+\frac{2}{n^2+2}+\cdots\cdots+\frac{n}{n^2+n}\right]$$
   (c) If $x_1=8$ and $x_{n+1}=\dfrac{1}{2}x_n+2$ for all $n\in N$, then show that the sequence $\{x_n\}$ is monotonically decreasing and bounded. Find limit. 3

7. (a) Use Cauchy's criterion of convergence to examine the convergence of the sequence $\{x_n\}$ where 2
   $$x_n=1+\frac{1}{1!}+\frac{1}{2!}+\cdots\cdots+\frac{1}{n!}$$
   (b) If the $n$-th term of the sequence $\{x_n\}$ is given by $x_n=\dfrac{n}{2}-\left[\dfrac{n}{2}\right]$, where $[x]$ is the greatest integer not greater than $x$, then find two subsequences of $\{x_n\}$, one of which converges to the upper limit and the other converges to the lower limit of $\{x_n\}$. 2
   (c) Show that every Cauchy sequence is bounded. Is the converse true? Give reasons in support of your answer. 2+2

8. (a) If $a_n > 0$ for all $n \in \mathbb{N}$ and if the sequence $(n^2 a_n)_n$ is convergent, show that the infinite series $\sum a_n$ is convergent. 2

(b) For any positive number $\alpha$, apply Cauchy's root test to check the convergence of the series $\sum a_n$ where for all $n \in \mathbb{N}$, 4

$$a_n = \left(1 + \frac{1}{n^\alpha}\right)^{-n^{\alpha+1}}$$

(c) Use the ratio test to check the convergence of the series 2

$$1 + \frac{3}{2!} + \frac{5}{3!} + \frac{7}{4!} + \dots\dots$$

9. (a) Let $(u_n)_n$ be a sequence of positive terms such that the infinite series $\sum u_n$ is convergent. Use comparison test to show that $\sum u_n^2$ is also a convergent series. 2

(b) Define absolute convergence of an infinite series of real numbers. Show that every absolutely convergent series is convergent. 1+2

(c) Use Leibnitz test to show that the alternating series $\sum (-1)^n \left[\sqrt{n^2+1} - n\right]$ is convergent. Show by comparison test (limit form) that this alternating series is not absolutely convergent. 1+2

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2022

## MTMACOR04T-MATHEMATICS (CC4)

### DIFFERENTIAL EQUATION AND VECTOR CALCULUS

Time Allotted: 2 Hours　　　　　　　　　　　　Full Marks: 50

*The figures in the margin indicate full marks.*
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*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) Show that the equation $\dfrac{dy}{dx}=\dfrac{1}{y}$ , $y(0)=0$ has more than one solution and indicate the possible reasons.

(b) Find all ordinary and singular points of the differential equation

$$2x^2\frac{d^2y}{dx^2}+7x(x+1)\frac{dy}{dx}-3y=0$$

(c) Solve: $\dfrac{dx}{dt}-7x+y=0$ ; $\dfrac{dy}{dt}-2x-5y=0$

(d) Reduce the equation $2x^2\dfrac{d^2y}{dx^2}+4y^2=x^2\left(\dfrac{dy}{dx}\right)^2+2xy\dfrac{dy}{dx}$ to Euler's homogeneous equation by the substitution $y=z^2$ .

(e) Show that if $y=y_1$ is a solution of $\dfrac{d^2y}{dx^2}+P\dfrac{dy}{dx}+Qy=0$, then another solution is $y=y_2$, where

$$y_2=y_1\int\frac{W(y_1,\, y_2)}{y_1^2}\,dx$$

$P$ and $Q$ being functions of $x$ and the Wronskian $W(y_1,\, y_2)$ satisfies the equation $\dfrac{dW}{dx}+PW=0$.

(f) If $\boldsymbol{u}=t\boldsymbol{i}-t^2\boldsymbol{j}+(t-1)\boldsymbol{k}$ and $\boldsymbol{v}=2t^2\,\boldsymbol{i}+6t\boldsymbol{k}$ , evaluate $\displaystyle\int_0^2(\boldsymbol{u}\times\boldsymbol{v})\,dt$ .

(g) Find the unit vector in the direction of the tangent at any point on the curve given by

$$\vec{r}=(a\ \cos t)\hat{i}+(a\ \sin t)\hat{j}+bt\,\hat{k}$$

(h) Find the volume of the parallelepiped whose edges are represented by

$$\boldsymbol{a} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k} \quad \text{and} \quad \boldsymbol{b} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k} \quad \text{and} \quad \boldsymbol{c} = 3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$$

(i) Find $\boldsymbol{r}$ from the equation $\dfrac{d^2r}{dt^2} = \boldsymbol{a}t + \boldsymbol{b}$, given that both $\boldsymbol{r}$ and $\dfrac{dr}{dt}$ vanish when $t = 0$.

2. (a) Find the necessary and sufficient condition that the three non-zero non-collinear vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ to be coplanar. 4

(b) If $\boldsymbol{a}$ and $\boldsymbol{b}$ be two non-collinear vectors such that $\boldsymbol{a} = \boldsymbol{c} + \boldsymbol{d}$, where $\boldsymbol{c}$ is a vector parallel to $\boldsymbol{b}$ and $\boldsymbol{d}$ is a vector perpendicular to $\boldsymbol{b}$, then obtain expressions for $\boldsymbol{c}$ and $\boldsymbol{d}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$. 4

3. (a) Prove that the necessary and sufficient condition that a vector function $\boldsymbol{f}(\boldsymbol{t})$ has a constant direction is $\boldsymbol{f} \times \dfrac{d\boldsymbol{f}}{dt} = \boldsymbol{0}$. 3

(b) (i) If $\boldsymbol{r} = (\cos nt)\boldsymbol{a} + (\sin nt)\boldsymbol{b}$, where $n$ is a constant, show that 3

$$\boldsymbol{r} \times \frac{dr}{dt} = n(\boldsymbol{a} \times \boldsymbol{b}) \quad \text{and} \quad \frac{d^2\boldsymbol{r}}{dt^2} + n^2\boldsymbol{r} = 0$$

(ii) If $\boldsymbol{r}(\boldsymbol{t}) = 5t^2\boldsymbol{i} + t\boldsymbol{j} - t^3\boldsymbol{k}$, then find the values of $\displaystyle\int_1^2 \left(\boldsymbol{r} \times \frac{d^2\boldsymbol{r}}{dt^2}\right) dt$. 2

4. (a) Prove that: $[\boldsymbol{a}+\boldsymbol{b} \quad \boldsymbol{b}+\boldsymbol{c} \quad \boldsymbol{c}+\boldsymbol{a}] = 2[\boldsymbol{a} \quad \boldsymbol{b} \quad \boldsymbol{c}]$ 4

(b) Show that the four points $4\boldsymbol{i} + 5\boldsymbol{j} + \boldsymbol{k}$, $-\boldsymbol{j} - \boldsymbol{k}$, $3\boldsymbol{i} + 9\boldsymbol{j} + 4\boldsymbol{k}$ and $4(-\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$ are coplanar. 4

5. (a) Reduce the equation 4

$$2x^2y\frac{d^2y}{dx^2} + ky^2 = x^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx}$$

to homogeneous form and hence solve it.

(b) Find the necessary and sufficient condition that the two solutions $y_1$ and $y_2$ of the equation 4

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$$

are linearly dependent.

6. (a) Solve the differential equation 4

$$\frac{d^2y}{dx^2} - 9y = x + e^{2x} - \sin 2x$$

by using the method by undetermined coefficients.

(b) Show that the equation 4

$$x^3 \frac{d^3y}{dx^3} - 6x\frac{dy}{dx} + 12y = 0$$

has three independent solutions of the form $y = x^r$, given that $y = x^2$ is a solution.

7. Solve:

(a) $(D^2 + 2D + 1)y = e^{-x} \log x$, (by the method of variation of parameters). 4

(b) $(D^2 - 1)y = x^2 \sin x$ 4

8. (a) Solve: $(D^2 + 4)x + y = te^{3t}$ ; $(D^2 + 1)y - 2x = \cos^2 t$ ; by operator method. 4

(b) Solve: $(D^4 - n^4)y = 0$ completely. Prove that if $Dy = y = 0$ when $x = 0$ and $x = l$, then 4

$$y = c_1(\cos nx - \cosh nx) + c_2(\sin nx - \sinh nx) \quad \text{and} \quad (\cos nl \, \cosh nl) = 1$$

9. (a) Obtain the power series solution of the differential equation 5

$$(1 - x^2)y'' + 2xy' - y = 0 \quad \text{about} \quad x = 0$$

(b) The equation of motion of a particle is given by 3

$$\frac{dx}{dt} + \omega y = 0 \quad ; \quad \frac{dy}{dt} - \omega x = 0$$

Find the path of the particle.

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 3rd Semester Examination, 2021-22

## MTMACOR05T-MATHEMATICS (CC5)

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Determine $f(1)$ so that the function $f(x)=\dfrac{x^2-1}{x-1}$ , $x\neq 1$ is continuous at $x=1$.

(b) If $f(x)=\begin{cases}1+x & , & x\le 2\\ 5-x & , & x>2\end{cases}$, then examine the existence of $f'(2)$.

(c) Show that $f(x)=x-[x]$ has a jump discontinuity at $x=1$.

(d) Show that $\frac{\sin x}{x}$ decreases steadily in $0<x<\frac{\pi}{2}$.

(e) Let $f:\mathbb{R}\longrightarrow\mathbb{R}$ be defined by

$$f(x)=\begin{cases}x\sin\frac{1}{x} & , & x\neq 0\\ 0 & , & x=0\end{cases}$$

Show that $f$ is continuous at $x=0$.

(f) Correct or justify:

Every bounded sequence is convergent.

(g) Check whether Rolle's theorem is applicable to the function $f(x)=|x|$ , $x\in[-1,1]$.

(h) Let $f(x)=(x-p)^n(x-q)^m$ in $[p,\, q]$. Show that there exists a $\xi\in(p,q)$ which divides $[p,\, q]$ in the ratio $n:m$.

(i) Show that for the function $f(x)=|x-1|$, $f'(1)$ does not exist.

2. (a) Let $f:[a,b]\longrightarrow\mathbb{R}$ be continuous in $[a,\, b]$. If $k\in\mathbb{R}$ satisfies $f(a)<k<f(b)$, then prove that there exists a point $c$ between $a$ and $b$ such that $f(c)=k$. 5

(b) Let $f:[a,b]\to[a,b]$ be a continuous function. Show that there exists at least one $c\in[a,b]$ such that $f(c)=c$. 3

3. (a) Let $A \subseteq \mathbb{R}$, $f : A \longrightarrow \mathbb{R}$ and $c \in A$. Let $f$ be continuous at $c$ and let $\{x_n\}$ be a sequence in $A$ such that $\lim_{n\to\infty} x_n = c$. Then show that $\lim_{n\to\infty} f(x_n) = f(c)$. 4

(b) A function is defined on $\mathbb{R}$ by 4

$$\begin{aligned} f(x) &= 1 \quad , \quad x \in \mathbb{Q} \\ &= 0 \quad , \quad x \in \mathbb{R} - \mathbb{Q} \end{aligned}$$

where $\mathbb{Q}$ is set of rational numbers. Prove that $f$ is continuous at no point $c \in \mathbb{R}$.

4. (a) Give an example with proper justifications to show that a bounded function on a closed and bounded interval need not be continuous. 3

(b) Show that the function 5

$$f(x) = x^2 \quad \forall\, x \in \mathbb{R}$$

is not uniformly continuous on $\mathbb{R}$ but its restriction to any non empty bounded interval $J$ of $\mathbb{R}$ is uniformly continuous.

5. (a) Let $f : I \to J$ be a bijective function where $I$, $J$ are intervals in $\mathbb{R}$. Let $f$ be differentiable at $d \in I$ and let $f'(d) \neq 0$. Show that $f^{-1}$ is differentiable at $f(d)$ and $(f^{-1})'[f(d)] = [f'(d)]^{-1}$. 4

(b) Let $f : I \longrightarrow \mathbb{R}$ be a function differentiable at $c \in I$, where $I$ is an interval in $\mathbb{R}$. Let $f'(c) > 0$. Show that there is a $\delta > 0$ so that 4

$$x \in (c-\delta, c) \cap I \Rightarrow f(x) < f(c)$$
$$x \in (c, c+\delta) \cap I \Rightarrow f(x) > f(c)$$

6. (a) Show that between any two distinct real roots of $e^x \sin x - 1 = 0$ there is at least one real root of $e^x \cos x + 1 = 0$. 3

(b) State and prove Lagrange's mean value theorem. 1+4

7. (a) Let $A \subseteq \mathbb{R}$ and $f, g, h : A \longrightarrow \mathbb{R}$ and $c \in \mathbb{R}$ be a limit point of $A$. 4

If $f(x) \leq g(x) \leq h(x)$ $\forall\, x \in A$, $x \neq c$ and if $\lim_{x\to c} f(x) = L = \lim_{x\to c} h(x)$ then show that $\lim_{x\to c} g(x) = L$.

(b) Let $g : \mathbb{R} \longrightarrow \mathbb{R}$ be defined by 4

$$g(x) = \begin{cases} x + 2x^2 \sin\frac{1}{x} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$$

Show that $g$ is not monotonic in any neighbourhood of zero.

8. (a) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be defined by 4

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x^2} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$$

Show that $f$ is differentiable on $\mathbb{R}$ but $f'$ is not continuous on $\mathbb{R}$.

(b) Let $I$ be an integral and a function $f : I \longrightarrow \mathbb{R}$ be differentiable at $c \in I$. Then show that $f$ is continuous at $c$. Is the converse true? Justify your answer. 4

9. (a) Show that $\frac{\tan x}{x} > \frac{x}{\sin x}$ , $0 < x < \frac{\pi}{2}$. 5

(b) Let $f : [0, 1] \longrightarrow \mathbb{R}$ be a continuous function which is differentiable on (0, 1). Show that the equation 3

$$f(1) - f(0) = \frac{f'(x)}{3x^2}$$

has at least one solution in (0, 1).

10.(a) Write with proper justification, Maclaurin's infinite series expansion for 4

$$f(x) = \sin x \ , \quad x \in \mathbb{R}$$

(b) Find the maximum and minimum values of $y = \sin x(1 + \cos x)$ , $0 \le x \le 2\pi$. 4

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 3rd Semester Examination, 2021-22

## MTMACOR07T-MATHEMATICS (CC7)

Time Allotted: 2 Hours | Full Marks: 40

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *four* questions from the rest**

1. Answer any ***four*** questions from the following: $2\times 4 = 8$

(a) Given $f(0)=3$, $f(1)=12$, $f(2)=81$, $f(3)=200$, $f(4)=100$, $f(5)=8$. Find $\Delta^5 f(0)$.

(b) If $u = xyz^2$ and errors in $x$, $y$, $z$ are 0.005, 0.002, 0.001 respectively at $x=3$, $y=z=1$, compute the maximum absolute error in evaluating $u$.

(c) Show that $\sum_{k=0}^{n-1} \Delta^2 f_k = \Delta f_n - \Delta f_0$.

(d) State Newton-Gregory's backward interpolation formula with its remainder term.

(e) Find an iterative formula to obtain the cube root of a positive number $N$.

(f) Find the solution of the differential equation

$$\frac{dy}{dx} = 1 + y \quad , \quad y(0) = 0$$

for $x = 0.2$ by using Euler's method (take step length $h = 0.1$).

(g) Find the value of $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\,dx$ by Trapezoidal rule.

2. (a) If $u = 4x^2y^3/z^4$ and error in $x$, $y$, $z$ be 0.001, compute the relative maximum error in $u$ when $x=y=z=1$. 4

(b) Find $y(3)$ from the following data: 4

$$y(0)=1\,,\ \ y(1)=3\,,\ \ y(2)=9\,,\ \ y(4)=81$$

3. (a) Find a real root of $x^3 - x = 1$ lying between 1 and 2 by Bisection method. Compute 6 iterations. 3

(b) Write down the geometrical interpretation of the Newton-Raphson method. 2

(c) Derive the convergence condition for Newton-Raphson method. 3

4. (a) Derive Newton's Backward interpretation formula. 4

(b) Use Stirling's formula to find $y(28)$, given that 4

$$y(20)=49225\,,\ \ y(25)=48316\,,\ \ y(30)=47236\,,\ \ y(35)=45926\,,\ \ y(40)=44306$$

5. Find the inverse of the matrix 8

$$A=\begin{pmatrix} 5 & 2 & 1 \\ 2 & 1 & 3 \\ 3 & 3 & 2 \end{pmatrix}$$

using LU decomposition method and hence solve the system of equations

$$5x+2y+z=12$$
$$2x+y+3z=13$$
$$3x+3y+2z=15$$

6. (a) A train is moving at the speed of 30 km/sec. Suddenly breaks are applied. The speed $v$ of the train per second after $t$ seconds is given by: 4

| Time ($t$) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
|---|---|---|---|---|---|---|---|---|---|---|
| Speed ($v$) | 30 | 24 | 19 | 16 | 13 | 11 | 10 | 8 | 7 | 5 |

Apply Simpson's $3/8^{\text{th}}$ rule to determine the distance moved by the train in 45 seconds.

(b) Write an algorithm to find the sum of only even numbers out of first $N$ numbers input by the user. 4

7. (a) Apply Euler's method to the initial value problem $\dfrac{dy}{dx}=x+y$ , $y=0$ when $x=0$, at $x=0$ to $x=1.0$ taking $h=0.2$. 4

(b) Deduce numerical differentiation formula based on Lagrange's interpolation formula. 4

8. (a) Compute the values of the unknown in the system of equations by Gauss-Jordan method. 5

$$x_1+3x_2+2x_3=17$$
$$x_1+2x_2+3x_3=16$$
$$2x_1-x_2+4x_3=13$$

(b) Prove that the $n$-th divided difference can be expressed as the quotient of two determinants of order $(n+1)$. 3

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2022

## MTMACOR08T-MATHEMATICS (CC8)

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Find the lower and upper integrals of the function.

$$f(x)=\begin{cases}1 & ; \quad x\in\mathbb{Q}\\ 0 & ; \quad x\notin\mathbb{Q}\end{cases}$$

(b) Find the Cauchy Principal Value of $\int_{-1}^{1}\frac{dx}{x^5}$.

(c) Test the convergence of $\int_{0}^{2}\frac{\log x}{\sqrt{2-x}}\,dx$.

(d) Show that $B(m,\ n)=B(n,\ m)$, for $m, n>0$.

(e) Examine whether the sequence of functions $\{f_n\}$ converges uniformly on $\mathbb{R}$, where for all $n\in\mathbb{N}$,

$$f_n(x)=\frac{n}{x+n}\quad,\quad x\in\mathbb{R}$$

(f) Find the limit function $f(x)$ of the sequence $\{f_n\}$ on $[0, \infty)$, where for all $n\in\mathbb{N}$,

$$f_n(x)=\frac{x^n}{1+x^n}\quad,\quad x\geq 0$$

Hence, state with reason whether $\{f_n\}$ converges uniformly on $[0, \infty)$.

(g) Show that the series $\sum_{n=1}^{\infty}\frac{n^5+1}{n^7+3}\left(\frac{x}{2}\right)^n$ is uniformly convergent on [–2, 2].

(h) Find the radius of convergence of the power series: $\sum(-1)^{n-1}x^n$

2. (a) For bounded function $f$ defined on an interval $[a, b]$ and any two partitions $P_1, P_2$ of $[a, b]$ show that $L(f, P_1)\leq U(f, P_2)$. 4

(b) Prove that a continuous function $f$ defined on a closed interval $[a, b]$ is integrable in the sense of Riemann. 4

3. (a) A function $f:[0,1]\to\mathbb{R}$ is defined by 2+2

$$f(x)=\frac{1}{3^n}\quad,\quad \frac{1}{3^{n+1}}<x\le\frac{1}{3^n}\quad,\quad n=0,1,2,\ldots\ldots$$
$$\quad = 0\quad,\qquad x=0$$

Show that $f$ is integrable in the sense of Riemann and $\int_0^1 f(x)\,dx=\frac{3}{4}$.

(b) Using Mean Value Theorem of Integral Calculus prove that 4

$$\frac{\pi^3}{24}\le\int_0^{\pi}\frac{x^2}{5+3\cos x}\,dx\le\frac{\pi^3}{6}$$

4. (a) Show that $\int_a^b (x-a)^{m-1}(b-x)^{n-1}\,dx=(b-a)^{m+n-1}\,\beta(m,n)\ ,\ m,n>0$. 4

(b) Test the convergence of the integral $\int_0^1\frac{\sqrt{x}}{e^{\sin x}-1}\,dx$. 4

5. (a) Let $f_n(x)=(x-[x])^n,\ x\in\mathbb{R},\ n\in\mathbb{N}$. Show that the sequence $\{f_n\}$ is convergent pointwise. Verify whether the convergence is uniform. 2+2

(b) If $\{f_n\}$ is a sequence of functions defined on a set $D$ converging uniformly to a function $f$ on $D$ such that each $f_n$ is continuous at some point $c\in D$, prove that $f$ is continuous at $c$. 4

6. (a) Verify the uniform convergence of the series 4

$$\sum_{n=0}^{\infty}\frac{x}{[(n+1)x+1][nx+1]}$$

on the interval $[a,b]$, where $0<a<b$.

(b) Show that the function $f(x)=\sum_{n=1}^{\infty}\frac{\sin nx}{n^3}$ is differentiable on $\mathbb{R}$. Find its derivative. 4

7. (a) If a series $\sum_{n=0}^{\infty}a_n x^n$ is convergent for some $x=a\ne 0$, then prove that the series converges absolutely for all $x$ with $|x|<|a|$. 3

(b) Find the radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{x^n}{2^n n^2}$. Using this, show that the series $\sum_{n=0}^{\infty}\frac{x^n}{2^{n+1}(n+1)}$ has the same radius of convergence. 3+2

8. (a) State Dirichlet's condition for convergence of a Fourier series. 2

(b) Obtain the Fourier series expansion of $f(x)$ in $[-\pi,\ \pi]$ where 4+2

$$f(x)=\begin{cases} 0 & , \quad -\pi \le x < 0 \\ \frac{1}{4}\pi x & , \quad 0 \le x \le \pi \end{cases}$$

Hence show that the sum of the series

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\cdots=\frac{\pi^2}{8}$$

9. (a) The function $f:[-2,\ 2] \to \mathbb{R}$ is defined by 4

$$\begin{aligned} f(x) &= x+1 \ , \quad -2 \le x \le 0 \\ &= x-1 \ , \quad 0 < x \le 2 \end{aligned}$$

Find the Fourier series of the function $f$.

(b) Expand the function $f(x)=x^2$, $0 < x \le \pi$ in a Fourier Sine series. 4

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2022

## MTMACOR09T-MATHEMATICS (CC9)

Time Allotted: 2 Hours Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) If $S$ be the set of all points $(x, y, z)$ in $\mathbb{R}^3$ satisfying the inequality $z^2 - x^2 - y^2 > 0$, determine whether or not $S$ is open. 2

(b) Show that the set $S = \{(x, y):\ x, y \in Q\}$ is not closed in $\mathbb{R}^2$. 2

(c) Prove / disprove: $S = \{(x, y) : |x| < 1,\ |y| < 1\}$ is open in $\mathbb{R}^2$. 2

(d) Show that $\lim\limits_{(x,y)\to(0,0)} (x+y) = 0$. 2

(e) If $u = F(y-z, z-x, x-y)$, then prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$. 2

(f) Find the gradient vector at each point at which it exists for the scalar field defined by $f(x, y, z) = x^2 - y^2 + 2z^2$. 2

(g) Use Stokes' theorem to prove that $\int\limits_C \vec{r}.d\vec{r} = 0$. 2

(h) What do you mean by conservative vector field? 2

2. (a) Show that the limit, when $(x, y) \to (0, 0)$ does not exist for $\lim \dfrac{2xy}{x^2 + y^2}$. 4

(b) If $f(x, y) = \sqrt{|xy|}$, find $f_x(0, 0)$, $f_y(0, 0)$. 2+2

3. (a) Show that the function $|x| + |y|$ is continuous, but not differentiable at the origin. 4

(b) Evaluate $\iint\limits_R (x + 2y)\, dx\, dy$, over the rectangle $R = [1, 2; 3, 5]$. 4

4. (a) For the function $f : D(\subset \mathbb{R}^2) \longrightarrow \mathbb{R}$ and $\beta$ be a unit vector in $\mathbb{R}^2$, define the directional derivative of $f$ in the direction of $\beta$ at the point $(a, b) \in \mathbb{R}^2$. Show that the directional derivative generalise the notion of partial derivatives. 4

(b) Prove that $f(x, y) = \{|x + y| + (x + y)\}^k$ is everywhere differentiable for all values of $k \geq 0$. 4

5. (a) Using divergence theorem evaluate $\iint\limits_S \mathbf{A}\cdot\mathbf{n}\,dS$, where $\mathbf{A}=(2x^2,\ y,\ -z^2)$ and $S$ denote the closed surface bounded by the cylinder $x^2+y^2=4$, $z=0$ and $z=2$. 4

(b) Find the directional derivative of $f(x,\ y)=2x^3-xy^2+5$ at (1, 1) in the direction of unit vector $\beta=\frac{1}{5}(3,\ 4)$. 4

6. (a) Show, by changing the order of integration, that $\int\limits_0^1 dx\int\limits_x^{1/x}\frac{y\,dy}{(1+xy)^2(1+y^2)}=\frac{\pi-1}{4}$. 4

(b) Show that $\iint\limits_E \frac{\sqrt{a^2b^2-b^2x^2-a^2y^2}}{\sqrt{a^2b^2+b^2x^2+a^2y^2}}\,dx\,dy=ab\frac{\pi}{4}\left(\frac{\pi}{2}-1\right)$, where $E$ is the region in the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 4

7. (a) Prove that of all rectangular parallelopiped of same volume, the cube has the least surface area, using Lagrange's multipliers method. 4

(b) If $z$ is a differentiable function of $x$ and $y$ and if $x=c\cosh u\,\cos v$, $y=c\sinh u\,\sin v$, then prove that 4

$$\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}=\frac{1}{2}e^2(\cosh 2u-\cos 2v)\left(\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}\right)$$

8. (a) Show that the vector field given by $A=(y^2+z^3,\ 2xy-5z\ ,\ 3xz^2-5y)$ is conservative. Find the scalar point function for the field. 4

(b) Evaluate $\int\limits_C (y\,dx+z\,dy+x\,dz)$, applying Stokes' Theorem, where $C$ is the curve given by $x^2+y^2+z^2-2ax-2ay=0$, $x+y=2a$ and begins at the point $(2a,\ 0,\ 0)$ and goes at first below the $z$-plane. 4

9. (a) Evaluate the line integral $\int\limits_C [2xy\,dx+(e^x+x^2)\,dy]$ by using Green's theorem, around the boundary $C$ of the triangle with vertices (0, 0), (1, 0), (1, 1). 4

(b) Find the surface area of the sphere $x^2+y^2+z^2=9$ lying inside the cylinder $x^2+y^2=3y$. 4

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2022

## MTMACOR10T-MATHEMATICS (CC10)

### RING THEORY AND LINEAR ALGEBRA-I

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Show that the ring $\left\{\begin{pmatrix} 2a & 0 \\ 0 & 2b \end{pmatrix} : a, b \in \mathbb{Z}\right\}$ does not contain unity.

(b) Solve $x^3 = x$ in the ring $\{\mathbb{Z}_6, +, .\}$ considering the equation over that ring.

(c) In the ring $R = \{f / f : [0,1] \to \mathbb{R}\}$ w.r.t. usual addition and multiplication of functions, show that, for any fixed point $c \in [0,1]$ the set $I_c = \{f \in R / f(c) = 0\}$ forms an ideal.

(d) Let $f : R \to S$ be a homomorphism from a ring $R$ to a ring $S$. Show that $f(-a) = -f(a)\ \forall\, a \in R$.

(e) State First Isomorphism Theorem for Rings.

(f) Write down a basis of the vectorspace $\mathbb{R}^3$ over $\mathbb{R}$, containing $(2, 3, 4)$ as a basis vector.

(g) Examine if $\{(x, y) \in \mathbb{R}^2 : x^2 + y = 0\}$ is a subspace of the vectorspace $\mathbb{R}^2$ over $\mathbb{R}$.

(h) Examine if $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x + y, x - y)\ \forall\, (x, y) \in \mathbb{R}^2$ is a linear transformation from the vectorspace $\mathbb{R}^2$ over $\mathbb{R}$ to itself.

2. (a) Find the units and the nonzero divisors of zero in the ring $\{\mathbb{Z}_{12}, +, .\}$ 2+2

(b) Examine if the ring $\left\{\begin{pmatrix} a & b \\ 2b & a \end{pmatrix} : a, b \in \mathbb{R}\right\}$ is a field. 4

3. (a) Show that the ring $C[0,1] = \{f / f : [0,1] \to \mathbb{R} \text{ continuum}\}$ is a ring with unity. Is $C[0,1]$ an integral domain? Justify. 2+2

(b) Show that the intersection of two ideals of a ring is an ideal of that ring but union of two ideals of a ring may not be an ideal of that ring. 2+2

4. (a) Suppose that $\{R, +, .\}$ is a ring with the property $a \cdot a = a \;\; \forall\, a \in R$. Show that $R$ is commutative and every element in $R$ is self-inverse w.r.t. '+'. 2+2

(b) Show that the field $\mathbb{Q}$ has no proper subfield. 2

(c) Find all units of $\mathbb{Z}\,[i]$. 2

5. Determine all possible ring homomorphisms from 2+2+2+2

(a) $\mathbb{Z} \to \mathbb{Z}$

(b) $\mathbb{Z}_3 \to \mathbb{Z}_6$

(c) $\mathbb{Z}_6 \to \mathbb{Z}_3$

(d) $\mathbb{Z} \to \mathbb{Z}_6$

6. (a) In the ring $\mathbb{Z}_{24}$, show that $I = \{[0], [8], [16]\}$ is an ideal. Find all elements of the quotient ring $\mathbb{Z}_{24}/\,I$. 4

(b) Define linearly independent set in a vectorspace $V$ over $\mathbb{R}$ and show that any nonempty subset of a linearly independent set in a vectorspace $V$ over $\mathbb{R}$ is again linearly independent. 2+2

7. (a) Show that $S = \{(x, y, z) \in \mathbb{R}^3 / \, x + 2y + z = 0 \text{ and } 2x + y + 3z = 0\}$ is a subspace of the vectorspace $\mathbb{R}^3$ over $\mathbb{R}$ and find a basis of $S$. 2+2

(b) Determine all possible subspaces of the vectorspaces $\mathbb{R}^3$ over $\mathbb{R}$ and $\mathbb{R}^2$ over $\mathbb{R}$. 2+2

8. Let $V$ and $W$ be vectorspaces over $\mathbb{R}$ and $T : V \to W$ be a linear transformation.

(a) Define kernel of $T$. 2

(b) Show that $\ker T$ is singleton set iff $T$ is injective and in this case, image of any linearly independent subset of $V$ is a linearly independent subset of $W$. 2+2+2

9. (a) Show that a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is injective iff it is surjective. 2+2

(b) Show that the function $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x - y, x + 2y, y + 3z)$ $\forall\, (x, y, z) \in \mathbb{R}^3$ is an invertible linear transformation and verify whether $T^{-1}(x, y, z) = \left(\dfrac{2x + y}{3}, \dfrac{y - x}{3}, \dfrac{x - y + 3z}{9}\right) \;\; \forall\, (x, y, z) \in \mathbb{R}^3$. 2+2

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

## MTMACOR14T-MATHEMATICS (CC14)

## RING THEORY AND LINEAR ALGEBRA II

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: 2×5 =10

(a) Let $f(x)=x^4+x^3-3x^2-x+2$ and $g(x)=x^4+x^3-x^2+x-2$. Find the gcd of $f(x)$ and $g(x)$, as polynomials over $\mathbb{Q}$.

(b) Let $f(x)=x^6+x^3+1\in\mathbb{Z}[x]$. Show that $f(x)$ is irreducible over $\mathbb{Q}$.

(c) Is $\mathbb{Z}[\sqrt{-5}]$ a UFD? Justify.

(d) Let $\beta=\{(2,1),(3,1)\}$ be an ordered basis for $\mathbb{R}^2$. Suppose that the dual basis of $\beta$ is $\beta^*=\{f_1, f_2\}$. Find $f_1(x, y)$ and $f_2(x, y)$.

(e) Consider the matrix $\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$ in $M_{2\times 2}(\mathbb{R})$. Is the given matrix diagonalizable? Justify.

(f) In an inner product space $V$, show that $\|x+y\|^2+\|x-y\|^2=2\|x\|^2+2\|y\|^2$, $\forall\, x, y\in V$.

(g) Let $V$ be an inner product space and let $T$ be a normal operator on $V$. Show that $\|T(x)\|=\|T^*(x)\|$, $\forall\, x\in V$.

(h) Show that any orthonormal set of vectors in an inner product space $V$ is linearly independent.

2. (a) Let $R$ be a UFD and $f(x), g(x)$ be two primitive polynomials in $R[x]$, then prove that $f(x)\,g(x)$ is also a primitive polynomial. 4

(b) Let $R$ be the ring $\mathbb{Z}\times\mathbb{Z}$. Solve the polynomial equation $(1,1)x^2-(5,14)x+(6,33)=(0,0)$ over $R$. Show that the linear equation $(5,0)x+(20,0)=(0,0)$ has infinitely many roots in $R$. 2+2

3. (a) Let $R$ be a ring with unity. Show that $R[x]/\langle x\rangle\simeq R$. 4

(b) Let $R$ be a principal ideal domain and $p\in R$. Show that $p$ is irreducible if and only if $p$ is prime. 4

4. (a) Let $f(x) \in F[x]$ be a polynomial of degree 2 or 3, where $F$ is a field. Show that $f(x)$ is irreducible over $F$ if and only if $f(x)$ has no zero in $F$. 4

(b) Show that $f(x) = x^4 - 2x^3 + x + 1$ is irreducible in $\mathbb{Q}[x]$. 4

5. (a) Let $V$ be an $n$-dimensional inner product space and $W$ be a subspace of $V$. Then prove that $\dim(V) = \dim(W) \oplus \dim(W^{\perp})$, where $W^{\perp}$ denotes the orthogonal complement of $W$. 5

(b) Define $T : P_1(\mathbb{R}) \to \mathbb{R}^2$ by $T(p(x) = (p(0), p(2))$, where $P_1(\mathbb{R})$ is the polynomials of degree atmost 1 over $\mathbb{R}$. Let $\beta$ and $\gamma$ be the standard ordered bases for $P_1(\mathbb{R})$ and $\mathbb{R}^2$ respectively. Find $[T]_{\beta}^{\gamma}$ and $[T^t]_{\gamma^*}^{\beta^*}$. Also show that $[T^t]_{\gamma^*}^{\beta^*} = ([T]_{\beta}^{\gamma})^t$. 3

6. Let $T$ be the linear operator on $P_2(\mathbb{R})$ defined by $T(f(x)) = f(x) + (x+1)f'(x)$ and let $\beta$ be the standard ordered basis for $P_2(\mathbb{R})$ and let $A = [T]_{\beta}$. Find the eigen values and the eigen vectors of $T$. Examine whether $T$ is diagonizable or not. 2+4+2

7. (a) Let $T$ be a linear operator on $\mathbb{R}^4$ defined by
$T(a, b, c, d) = (a + b + 2c - d, b + d, 2c - d, c + d)$ and let $W = \{(t, s, 0, 0) : t, s \in \mathbb{R}\}$. Show that $W$ is a $T$-invariant subspace of $\mathbb{R}^4$. 3

(b) Let $T$ be a linear operator on a vector space $V$, and let $\lambda_1, \lambda_2, \cdots, \lambda_k$ be distinct eigen values of $T$. If $v_1, v_2, \cdots, v_k$ are eigen vectors of $T$ such that $\lambda_i$ corresponds to $v_i (1 \le i \le k)$, then show that $\{v_1, v_2, \cdots, v_k)$ is linearly independent. 5

8. (a) Let $T$ be a linear operator on a finite dimensional vector space $V$ and let $f(t)$ be the characteristic polynomial of $T$. Then prove that $f(T) = T_0$, where $T_0$ denotes the zero transformation. 4

(b) Let $\langle \, , \rangle$ be the standard inner product on $\mathbb{C}^2$. Prove that there is no nonzero linear operator on $\mathbb{C}^2$ such that $\langle \alpha, T\alpha \rangle = 0$ for every $\alpha$ in $\mathbb{C}^2$. Generalize this result for $\mathbb{C}^n$, where $n$ is any positive integer greater equal to 2. 4

9. (a) Apply Gram-Schmidt process to the subset
$S = \{(2, -1, -2, 4), (-2, 1, -5, 5), (-1, 3, 7, 11)\}$ of the inner product space $\mathbb{R}^4$ to obtain an orthogonal basis for span$(S)$. Then normalize the vectors in this basis to obtain an orthonormal basis $\beta$ for span$(S)$. 5

(b) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear operator whose matrix representation in the standard ordered basis is given by 3

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \; 0 < \theta < \pi.$$

Show that $T$ is normal.

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

## MTMADSE01T-MATHEMATICS (DSE1/2)

### LINEAR PROGRAMMING

Time Allotted: 2 Hours  Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) Why we introduce artificial variable in Charne's penalty method?

(b) Define extreme point of a convex set. Give an example of a convex set having no extreme point.

(c) Find in which half space of the hyperplane $2x_1+3x_2+4x_3-x_4=6$, the points. $(4,-3,2,1)$ and $(1,2,-3,1)$ lie.

(d) Prove that the solution of the transportation problem is never unbounded.

(e) Solve the following $2\times2$ game problem by algebraic method:

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \end{array}$$

(f) Find graphically the feasible space, if any for the following

$2x_1+x_2\le 6$

$5x_1+3x_2\ge 15$, $x_1, x_2\ge 0$

(g) Prove that if the dual problem has no feasible solution and the primal problem has a feasible solution, then the primal objective function is unbounded.

(h) Find the optimal strategies and game value of the following game problem.

Player B

Player A

| | |
|---|---|
| 9 | 5 |
| 7 | 11 |

(i) Suppose you have a linear programming problem with five constraints and three variables. Then what problem, primal or dual will you select to solve? Give reasons.

2. (a) Solve graphically the L.P.P. 4

Maximize $z = 5x_1 - 2x_2$

Subject to $5x_1 + 6x_2 \geq 30$

$9x_1 - 2x_2 = 72$

$x_2 \leq 9$

$x_1, x_2 \geq 0$

(b) Show that the L.P.P. 4

Maximize $z = 4x_1 + 14x_2$

Subject to $2x_1 + 7x_2 \leq 21$

$7x_1 + 2x_2 \leq 21$

$x_1\ x_2 \geq 0$

admits of an infinite number of solutions.

3. Use Charne's Big-M method to solve the L.P.P. 8

Minimize $z = 2x_1 + x_2$

Subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$, $x_1, x_2 \geq 0$

4. (a) Let $x$ be any feasible solution to the primal problem and $v$ be any feasible solution to its dual problem then prove that $cx \leq b^T v$. 4

(b) Find the dual of the following problem 4

Maximize $Z = 2x_1 + 3x_2 + 4x_3$

Subject to $x_1 - 5x_2 + 3x_3 = 7$

$2x_1 - 5x_2 \leq 3$

$3x_2 - x_3 \geq 5$

$x_1, x_2 \geq 0, x_3$ is unrestricted in sign.

5. (a) Prove that a subset of the columns of the coefficient matrix of a transportation problem are linearly dependent if the corresponding cells or a subset of them can be sequenced to form a loop. 4

(b) Using North-West corner rule find the initial basic feasible solution of the following transportation problem hence find the optimal solution. 4

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $a_j$ |
|---|---|---|---|---|---|
| $O_1$ | 2 | 1 | 3 | 4 | 30 |
| $O_2$ | 3 | 2 | 1 | 4 | 50 |
| $O_3$ | 5 | 2 | 3 | 8 | 20 |
| $b_j$ | 20 | 40 | 30 | 10 | |

6. (a) Prove that the dual of the dual is the primal. 3

   (b) Find the optimal assignment and minimum cost for the assignment problem with the following cost matrix: 5

| | $V$ | $W$ | $X$ | $Y$ | $Z$ |
|---|---|---|---|---|---|
| $A$ | 3 | 5 | 10 | 15 | 8 |
| $B$ | 4 | 7 | 15 | 18 | 8 |
| $C$ | 8 | 12 | 20 | 20 | 12 |
| $D$ | 5 | 5 | 8 | 10 | 6 |
| $E$ | 10 | 10 | 15 | 25 | 10 |

7. (a) In a two persons zero sum game, if the $2\times 2$ pay-off matrix has no saddle point then find the game value and optimal mixed strategies for the two players. 4

   (b) Solve graphically the following game problem: 4

| | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|---|---|---|---|---|
| $A_1$ | 1 | 2 | 6 | 12 |
| $A_2$ | 8 | 6 | 3 | 2 |

8. (a) Show that every finite two person zero sum game can be expressed as a linear programming problem. 4

   (b) Solve the following game problem by converting it into a L.P.P.: 4

Player Q

| | | $Q_1$ | $Q_2$ | $Q_3$ |
|---|---|---|---|---|
| | $P_1$ | 4 | 2 | 5 |
| Player P | $P_2$ | 2 | 5 | 1 |
| | $P_3$ | 5 | 1 | 6 |

9. (a) In a rectangular game, the pay-off matrix $A$ is given by 6

$$A=\begin{pmatrix} 3 & 2 & -1 \\ 4 & 0 & 5 \\ -1 & 3 & -2 \end{pmatrix}$$

   state, giving reason whether the players will use pure or mixed strategies. What is the value of the game?

   (b) Let $(a_{ij})_{m\times n}$ be the pay-off matrix for a two person zero-sum game. Then prove that 2

$$\max_{1\le i\le m}\left[\min_{1\le j\le n}\{a_{ij}\}\right]\le \min_{1\le j\le n}\left[\max_{1\le i\le m,}\{a_{ij}\}\right]$$

10.(a) Solve the following game by graphical method 4

$$\text{Player A} \quad \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{array} \overset{\text{Player B}}{\begin{array}{cc} B_1 & B_2 \\ \hline \end{array}} \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix}$$

(b) Prove that if a fixed number be added to each element of a pay-off matrix of a rectangular game, then the optimal strategies remain unchanged while the value of the game will be increased by that number. 4

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

## MTMADSE02T-MATHEMATICS (DSE1/2)

### NUMBER THEORY

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) If $\phi$ denotes the Euler's phi function, then prove that $\phi(n) \equiv 0 \pmod 2$, $\forall\, n \geq 3$.

(b) Solve $140x \equiv 133 \pmod{301}$.

(c) Check if Goldbach's conjecture is true for $n = 2022$.

(d) If $n$ has a primitive root, prove that it has exactly $\phi(\phi(n))$ primitive roots.

(e) Find all solutions to the Diophantine equation $24x + 138y = 18$.

(f) In RSA encryption, is $e = 20$, a valid choice for $N = 11 \times 13$?

(g) List down the quadratic non-residues in $\mathbb{Z}_{10}^{*}$, with proper explanation.

(h) Prove that $(p-2)! \equiv 1 \pmod p$, where $p$ is a prime.

(i) Find the number of positive divisors of $2^{2020} \times 3^{2021}$.

2. (a) If $f$ is a multiplicative function and $F$ is defined as $F(n) = \sum_{d \mid n} f(d)$, then prove $F$ to be multiplicative as well. 5

(b) Prove that there exists a bijection between the set of positive divisors of $p_1^{\alpha}$ and $p_2^{\beta}$, if and only if $\alpha = \beta$, where $p_1$ and $p_2$ are distinct primes. 3

3. (a) For each positive integer $n$, show that 3

$$\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$$

(b) Let $x$ and $y$ be real numbers. Prove that the greatest integer function satisfies the following properties: 3+2

(i) $[x+n] = [x] + n$ for any integer $n$

(ii) $[x] + [-x] = 0$ or $-1$ according to $x$ is an integer or not

4. (a) Solve the congruence $72x \equiv 18 \pmod{42}$. 5

(b) Let $a$, $b$ and $m$ be integers with $m > 0$ and $\gcd(a, m) = 1$. Then prove that the congruence $ax \equiv b \pmod{m}$ has a unique solution. 3

5. (a) Prove that, in $\mathbb{Z}_n^*$, the set of all quadratic residues form a subgroup of $\mathbb{Z}_n^* = \mathbb{Z}_n \setminus \{\bar{0}\}$. 4

(b) Prove that $\mathbb{Z}_{15}^*$ is not cyclic where $\mathbb{Z}_n^*$ is the collection of units in $\mathbb{Z}_n$. 4

6. (a) Suppose, $c_1$ and $c_2$ are two ciphertexts of the plaintexts $m_1$ and $m_2$ respectively, in an RSA encryption, using the same set of keys. Prove that, $c_1c_2$ is an encryption of $m_1m_2$. 3

(b) Prove that, in RSA encryption, the public key may never be even. 3

(c) Find $\phi(2021)$. 2

7. (a) Prove that there are no primitive roots for $\mathbb{Z}_8^*$. 2

(b) Let $\bar{g}$ be a primitive root for $\mathbb{Z}_p^*$, $p$ being an odd prime. Prove that $\bar{g}$ or $\overline{g+p}$ is a primitive root for $\mathbb{Z}_{p^2}^*$. 6

8. (a) Prove that the Mobius $\mu$-function is multiplicative. 6

(b) State the Mobius inversion formula. 2

9. (a) Show that Goldbach Conjecture implies that for each even integer $2n$ there exist integers $n_1$ and $n_2$ with $\Phi(n_1) + \Phi(n_2) = 2n$. 4

(b) Prove that the equation $\Phi(n) = 2p$, where $p$ is a prime number and $2p+1$ is composite, is not solvable. 4

10.(a) Determine whether the following quadratic congruences are solvable: 2+2

(i) $x^2 \equiv 219 \pmod{419}$

(ii) $3x^2 + 6x + 5 \equiv 0 \pmod{89}$.

(b) Show that 7 and 18 are the only incongruent solutions of 4

$$x^2 \equiv -1 \pmod{5^2}$$

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

## MTMADSE03T-MATHEMATICS (DSE1/2)

### PROBABILITY AND STATISTICS

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* questions from the rest**

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Give axiomatic definition of probability.

(b) Consider an experiment of rolling two dice. Define a random variable over the event space of this experiment.

(c) Consider an experiment of tossing two coins. Find the probability of two heads given atleast one head.

(d) Prove that $P(B\,|\,A) \geq 1 - \dfrac{P(\overline{B})}{P(A)}$

(e) Distribution function $F(x)$ of a random variable $X$ is given by

$$\begin{aligned} F(x) &= 1 - \frac{1}{2}e^{-x},\ x \geq 0 \\ &= 0, \text{ elsewhere} \end{aligned}$$

Find $P(X = 0)$ and $P(X > 1)$.

(f) The joint density function of $X$ and $Y$ is given by

$$f(x, y) = \begin{cases} k(x+y), & x > 0,\ y > 0,\ x + y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find the value of $k$.

(g) Two random variables $X$ and $Y$ have zero means and standard deviations 1 and 2 respectively. Find the variance of $X + Y$ if $X$ and $Y$ are uncorrelated.

(h) State Tchebycheff's inequality.

(i) Explain what are meant by a statistic and its sampling distribution.

2. (a) Two cards are drawn from a well-shuffled pack. Find the probability that at least one of them is a spade. 3+5

(b) Obtain the Poisson approximation to the binomial law, on stating the assumption made by you.

3. A random variable $X$ has the following probability distribution: 2+(3+3)

| $x_i$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $f_i$ | 0 | $k$ | $2k$ | $2k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2+k$ |

(a) Find $k$

(b) Evaluate $P(X<6)$, $P(X \geq 6)$

4. (a) Let $f(x, y)$ be the joint p.d.f. of $X$ and $Y$. Prove that $X$ and $Y$ are independent if and only if $f(x, y) = f_x(x)\ f_y(y)$. 4+4

(b) Write down the distribution function $\phi(x)$ of a standard normal distribution and prove that $\phi(0) = \frac{1}{2}$.

5. (a) If $X$ be a $\gamma(l)$ variate, find $E\{\sqrt{X}\}$ 2+(3+3)

(b) Find the mean and standard deviation of a binomial distribution.

6. (a) The joint density function of $X$ and $Y$ is given by 5+3

$$f(x, y) = \begin{cases} k(x+y), \text{ for } 0<x<1, 0<y<1 \\ 0, \qquad \text{elsewhere} \end{cases}$$

Find

(i) the value of $k$

(ii) the marginal density functions

(iii) the conditional density functions

Are $X$ and $Y$ independent?

(b) The joint density function of the random variable $X$, $Y$ is given by:

$$f(x, y) = 2\ (0<x<1, 0<y<x).$$

Compute $P\left(\frac{1}{4} < X < \frac{3}{4} \,\middle|\, Y = \frac{1}{2}\right)$

7. (a) If $\sigma_x^2, \sigma_y^2$ and $\sigma_{x-y}^2$ be the variances of $X$, $Y$ and $X-Y$ respectively, then prove that 3+3+2

$$\rho_{xy} = (\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2)/2\sigma_x\sigma_y .$$

(b) Find $k$ such that $\rho_{uv} = 0$ where $U = X + kY$ and $V = X + \frac{\sigma_x}{\sigma_y}Y$.

(c) If one of the regression coefficients is more than unity, prove that the other must have been less than unity.

8. (a) Define the concept of convergence in probability. If $X_n \xrightarrow[\text{in } p]{} X$, $Y_n \xrightarrow[\text{in } p]{} Y$, as $n \to \infty$, show that $X_n \pm Y_n \xrightarrow[\text{in } p]{} X \pm Y$ as $n \to \infty$ 5+3

(b) State Central Limit Theorem for independent and identically distributed random variable with finite variance.

9. Let $\mu = (\mu_1, \mu_2, \mu_3, \mu_4)$ be the stationary distribution for a Markov Chain on the state space $\{1, 2, 3, 4\}$ with transition probability matrix $P$. Suppose that the states 1 and 2 are transient and the states 3 and 4 form a communicating class. Which of the following are true? 8

(a) $\mu p^3 = \mu p^5$

(b) $\mu_1 = 0$ and $\mu_2 = 0$

(c) $\mu_3 + \mu_4 = 1$

(d) One of $\mu_3$ and $\mu_4$ is zero.

**OR**

(a) Prove that Central Limit Theorem (for equal components) implies Law of Large Numbers for equal components. 5+3

(b) A random variable $X$ has probability density function $12x^2(1-x), (0 < x < 1)$. Compute $P(|X - m| \geq 2\sigma)$ and compare it with the limit given by Tchebycheff's inequality.

10.(a) Find the maximum likelihood estimate of $\sigma^2$ for a normal $(m, \sigma)$ population if $m$ is known. 4+4

(b) The wages of a factory's workers are assumed to be normally distributed with mean $m$ and variance 25. A random sample of 25 workers gives the total wages equal to 1250 units. Test the hypothesis $m = 52$ against the alternative $m = 49$ at 1% level of significance.

$$\left[\frac{1}{2\pi}\int_{-\infty}^{2.32} e^{-\frac{x^2}{2}}\,dx = 0.01\right]$$

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

## MTMADSE04T-MATHEMATICS (DSE3/4)

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) If $a$ and $b$ are positive, prove that the equation $x^5-5ax+4b=0$ has three real roots or only one according as $a^5 >$ or $<$ $b^4$.

(b) Remove the second term of the equation $x^3+6x^2+12x-19=0$ and solve it.

(c) Examine whether $x^4-x^3+x^2+x-1=0$ is a reciprocal equation.

(d) If $\alpha$ be a root of the equation $x^3+3x^2-6x+1=0$, prove that the other roots are $\frac{1}{1-\alpha}$ and $\frac{\alpha-1}{\alpha}$.

(e) If $\alpha_1, \alpha_2, \cdots, \alpha_n$ be roots of the equation $x^n+nax+b=0$, prove that

$$(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)\cdots(\alpha_1-\alpha_n)=n(\alpha_1^{n-1}+a).$$

(f) Find the remainder when the polynomial $f(x)$ is divided by $(x-\alpha)(x-\beta)$, $\alpha \neq \beta$.

(g) Form a biquadratic equation with real coefficients two of whose roots are $2i \pm 1$.

(h) If $\alpha(\neq 1)$ be any $n^{\text{th}}$ root of unity, then prove that the sum $1+3\alpha+5\alpha^2+\cdots$ upto $n^{\text{th}}$ term $=\frac{2n}{\alpha-1}$.

2. (a) Show that if the roots of the equation $x^4+x^3-4x^2-3x+3=0$ are increased by 2, the transformed equation is a reciprocal equation. Solve the reciprocal equation and hence obtain the solution of the given equation. 4

(b) Solve the equation $x^7-1=0$. Deduce that $2\cos\frac{2\pi}{7}$, $2\cos\frac{4\pi}{7}$, $2\cos\frac{8\pi}{7}$ are roots of the equation $t^3+t^2-2t-1=0$. 4

3. (a) If $\alpha$ is a special root of $x^{11}-1=0$, prove that $(\alpha+1)(\alpha^2+1)\cdots(\alpha^{10}+1)=1$. 4

(b) Applying Strum's theorem show that the equation $x^3-2x-5=0$ has one positive real root and two imaginary roots. 4

4. (a) If the equation $x^4 - 4px^3 + 8x^2 + 1 = 0$ has a multiple root $\lambda$, prove that $3p = \dfrac{\lambda^2 + 3}{\lambda}$ and the only positive value of $p$ is $\left(\dfrac{4}{3}\right)^{\frac{3}{4}}$. 4

(b) Show that the equation $x^4 - 14x^2 + 24x + k = 0$ has four real and unequal roots if $-11 < k < -8$. 4

5. (a) Find the condition that the roots of the equation $x^3 + 3Hx + G = 0$ may have three real and distinct roots. 4

(b) Find the upper limit of the real roots of the equation $x^4 - 5x^3 + 40x^2 - 8x + 24 = 0$. 4

6. (a) Applying Newton's theorem find the sum of 7th powers of the roots of the equation $x^3 + qx + r = 0$. 4

(b) Show that the cubes of the roots of the cubic $x^3 + ax^2 + bx + ab = 0$ are the roots of the cubic $x^3 + a^3x^2 + b^3x + a^3b^3 = 0$. 4

7. (a) Prove that the equation $(x+1)^4 = a(x^4 + 1)$ is a reciprocal equation if $a \neq 0$ and solve it when $a = -2$. 4

(b) Find the values of $a$ for which the equation $ax^3 - 6x^2 + 9x - 4 = 0$ may have multiple roots and solve the equation in each case. 4

8. (a) If $\alpha$ be a multiple root of order 3 of the equation $x^4 + bx^2 + cx + d = 0$, show that $\alpha = -\dfrac{8d}{3c}$. 4

(b) The equation $3x^4 + x^3 + 4x^2 + x + 3 = 0$ has four distinct roots of equal moduli. Solve it. 4

9. (a) If $\alpha, \beta, \gamma$ be the roots of the equation $x^3 - qx + r = 0$, find the equation whose roots are $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} - \dfrac{1}{\gamma^2}$, $\dfrac{1}{\beta^2} + \dfrac{1}{\gamma^2} - \dfrac{1}{\alpha^2}$, $\dfrac{1}{\gamma^2} + \dfrac{1}{\alpha^2} - \dfrac{1}{\beta^2}$. 4

(b) If $\alpha_1, \alpha_2, \cdots \alpha_n$ be the roots of the equation $x^n + \dfrac{x^{n-1}}{1!} + \dfrac{x^{n-2}}{2!} + \cdots + \dfrac{1}{n!} = 0$ and $S_r = \sum \alpha_1^r$, show that $S_r = 0$ for $r = 2, 3, \cdots n$ but $S_r \neq 0$ for $r = n+1, n+2, \cdots$ 4

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

## MTMADSE05T-MATHEMATICS (DSE3/4)

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Give an example of an order preserving map between two ordered sets.

(b) In any lattice $L$, prove that

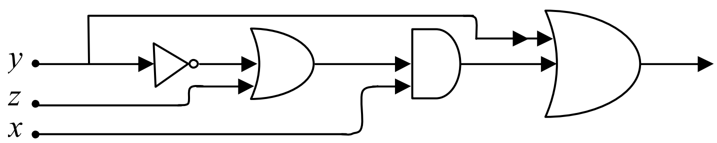
$$x\wedge(y\vee z)\geq(x\wedge y)\vee(x\wedge z),$$

for all $x, y, z\in L$.

(c) Use a Karnaugh-map to find the minimized sum-of-product Boolean expression of the Boolean expression

$$xyz+xyz'+xy'z'+x'yz+x'yz'.$$

(d) Write down the Boolean expression that represents the following logic-circuit.




(e) On the alphabet $\Sigma=\{0,1\}$, show that

$$1^*0+1^*0(\lambda+0+1)^*(\lambda+0+1)=1^*0(0+1)^*$$

where $\lambda$ is the empty string over $\Sigma$.

(f) Give state diagram of a DFA recognizing the following language over the alphabet $\{0,1\}:\{w\mid w$ is any string except 11 and 111$\}$.

(g) Draw a derivation tree that yields $a^4\in L(G)$, where $G=(\{s\},\{a,b\},S,P)$ is a context-free grammar with $P=\{S\to ss, S\to a\}$.

(h) Can a Turing machine contain just a single state? Give reasons.

2. (a) Define maximal and minimal elements in a poset. 1

(b) Show that any finite nonempty subset $X$ of a poset has minimal and maximal elements. 2

(c) Let $(P, \leq)$ be a finite poset. Show that the order $\leq$ can always be extended to a total order $\preceq$ on $P$, in the sense that, for all $x, y \in P$, $x \leq y \Rightarrow x \preceq y$. 2

(d) Using the result stated in (c), determine two total ordering relations on the set of positive divisors of 36 into which the order of the poset $D_{36}$ of divisors of 36 can be extended. 3

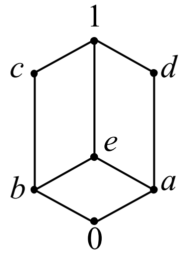
3. (a) For two lattices $L$ and $K$, prove that a mapping $\phi : L \to K$ is a lattice isomorphism if only if $\phi$ is an order isomorphism. 4

(b) In any lattice $L$, prove that the following identities are equivalent: 2

(i) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z), \ \forall x, y, z \in L$

(ii) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c), \ \forall a, b, c \in L$.

(c) The Hasse diagram given below represents a lattice: 2




Is this lattice distributive? Justify your answer with proper reason.

4. (a) Define a modular lattice. 1

(b) Show that every distributive lattice is modular. 1

(c) Draw the Hasse diagram for a pentagon $N_5$ of five elements. Show that the lattice $N_5$ is non-modular. 3

(d) Let $L$ be a lattice such that none of its sublattices is isomorphic to a pentagon. Prove that $L$ is a modular lattice. 3

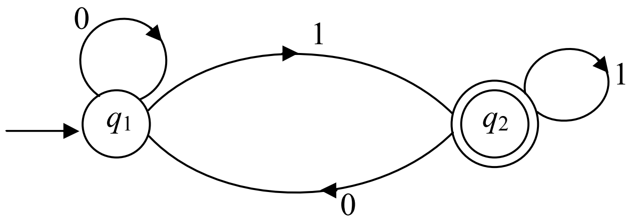
5. (a) Find the essential prime implicants of the Boolean function $f(A, B, C, D) = \sum m(1, 5, 6, 12, 13, 14)$. Hence find the minimal expression for $f(A, B, C, D)$ by using Quine-McClusky method. 3+3

(b) Find the Boolean expression in CNF which generates the following truth function: 2

| $x_1$ | $x_2$ | $x_3$ | $f(x_1, x_2, x_3)$ |
|---|---|---|---|
| 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

6. (a) Let $R \subseteq \Sigma^*$ and $\lambda \notin R$; where $\lambda$ is the empty string. For any $S \subseteq \Sigma^*$, prove that $S = SR$ if and only if $S = \phi$. 2+2

   (b) Consider the binary alphabet $\Sigma = \{0, 1\}$. Determine the regular expression for the language recognized by the DFA, $M$ whose transition graph is as follows: 4




7. (a) Use the pumping lemma for context free languages to show that the language $B = \{a^n b^n c^n \mid n \geq 0\}$ is not context-free. 4

   (b) Let $M = (S, \Sigma, \delta, q_0, F)$ be a non-deterministic finite automaton, in which $S = \{q_0, q_1, q_2\}$, $\Sigma = \{0, 1\}$, $F = \{q_2\}$ and the transition function $\delta$ is given by the following transition table: 4

| $\delta$ | 0 | 1 |
|---|---|---|
| $\rightarrow q_0$ | $\{q_0\}$ | $\{q_0, q_1\}$ |
| $q_1$ | $\phi$ | $\{q_2\}$ |
| $q_2$ | $\phi$ | $\phi$ |

   Construct a three-state DFA, $M_1$ equivalent to NFA, $M$. Also draw the transition graph of the DFA, $M_1$.

8. (a) Define Chomsky normal form of context-free grammar. 1

   (b) Transform the grammar with productions 3

$$S \rightarrow aSaaA \mid A$$
$$A \rightarrow abA \mid bb$$

   into Chomsky normal form.

   (c) Let $L_1$ be a context-free language and $L_2$ be a regular language. Prove that $L_1 \cap L_2$ is a context-free language. 4

9. Show that the collection of (Turing) decidable languages is closed under the operation of 2+2+2+2
   (i) union
   (ii) concatenation
   (iii) star
   (iv) complementation.

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

## MTMADSE06T-MATHEMATICS (DSE3/4)

### MECHANICS

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Find the equation of the line of action of the resultant of a system of coplanar forces having total moments $G, 2G, 3G$ about the points $(0, 0), (0, 1), (2, 4)$ respectively.

(b) What is meant by limiting friction? Why is it limiting?

(c) Find the centre of gravity of a surface revolving round the axis of $y$.

(d) Define Pointsot's central axis of a system of forces acting on a body.

(e) Are the centre of suspension and centre of oscillation of a compound pendulum reversible? Justify your answer.

(f) Find the degrees of freedom of three particles in a two-dimensional plane, two of which are connected by a fixed straight line.

(g) Define an apse and apsidal angle for a central orbit.

(h) What is the difference between a simple pendulum and a compound pendulum?

## UNIT-I

### ANALYTICAL STATICS

2. (a) Find the condition for the astatic equilibrium of a rigid body acted on by a system of coplanar forces. 4

(b) A square hole is punched out of a circular lamina, the diagonal of the square being a radius of the circle. Find the position of the centre of gravity of the remainder. 4

3. Forces $P, Q, R$ act along three straight lines given by the equation $y = 0$, $z = c$; $z = 0$, $x = a$; $x = 0$, $y = b$. Find the pitch of the equivalent wrench. 4+4

Also show that if the wrench reduces to a single force, then the line of action of the forces lies on the hyperboloid $(x-a).(y-b).(z-c) = xyz$.

4. (a) A solid frustum of paraboloid of height $h$ and latus rectum $4a$, rests with its vertex on the vertex of a paraboloid of revolution of latus rectum $4b$. Deduce the condition of stable equilibrium of the system. 4

(b) Two equal uniform rods $AB$ and $AC$, each of length $2l$ are freely jointed at $A$ and rest on a smooth vertical circle of radius '$a$'. Show that if the angle between the rods be $\frac{\pi}{2}$, then $l = 2a$. 4

## UNIT-II

### ANALYTICAL DYNAMICS

5. Derive the components of velocity and acceleration of a particle referred to a set of rotating rectangular axes. 4+4

6. (a) Find the condition that the orbit of a satellite will be an ellipse, parabola or a hyperbola. 4

(b) A particle describes an ellipse under a force $\frac{\mu}{(\text{distance})^2}$ towards a focus. If it was projected with velocity $V$ from a point at a distance $r$ from the centre of force, show that the periodic time is 4

$$\frac{2\pi}{\sqrt{\mu}}\left(\frac{2}{r}-\frac{V^2}{\mu}\right)^{-3/2}$$

7. (a) Deduce the differential equation of a central orbit under a central force in two-dimensional polar coordinates. 4

(b) A circular orbit of radius '$a$' is described under the central attractive force $f(r)=\mu\left(\frac{b}{r^2}+\frac{c}{r^4}\right)$, $\mu>0$. Deduce the condition of stability of the motion. 4

8. (a) Define momental ellipsoid and find it at the centre of an elliptic plate. 4

(b) Deduce the equation of motion of a rigid body from D'Alembert's principle. 4

9. (a) Show that the moment of inertia of elliptic area of mass $M$ and semi axes $a$ and $b$ about a diameter of length $r$ is $\frac{M}{4}.\frac{a^2b^2}{r^2}$. 5

(b) A fine string has two masses $M$ and $M'$ tied to its ends and passes over a rough pulley, of mass $m$ whose centre is fixed, if the string does slip over the pulley, show that $M$ will descend with acceleration $\frac{M-M'}{M+M'+mk^2/a^2}.g$ where $a$ is the radius and $k$ is the radius of gyration of the pulley. 3

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 1st Semester Examination, 2021-22

## MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)

### DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Does $\lim\limits_{(x,\,y)\to(0,\,0)} \dfrac{2xy^3}{x^2+y^6}$ exist? Give reasons. 2

(b) Use $\varepsilon$-$\delta$ definition of the limit to prove $\lim\limits_{x\to -3} x^2 = 9$. 2

(c) Find the coordinates of the points on the curve $y = x^3 - 6x + 7$ where the tangent is parallel to $y = 6x + 1$. 2

(d) Find domain of the function $f(x) = \sqrt{x-1} + \sqrt{5-x}$. 2

(e) Is Rolle's theorem applicable for the function $f(x) = x^2 - 5x + 6$ in [1, 4]? Justify your answer. 2

(f) Evaluate $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$. 2

(g) Prove that the function $f:\mathbb{R}\longrightarrow\mathbb{R}$ defined by $f(x) = \sin x,\ x\in\mathbb{R}$ is continuous on $\mathbb{R}$ by using the $\varepsilon$-$\delta$ definition of continuity. 2

(h) Examine the nature of discontinuity of the function $f$ defined by 2

$$f(x) = \begin{cases} \dfrac{1}{\sqrt{x}} & x > 0 \\ 0 & x = 0 \end{cases}$$

at 0.

(i) Find the curvature of the parabola $x^2 = 12y$ at the point $\left(-3, \frac{3}{4}\right)$. 2

2. (a) A function $f$ in [0, 1] is defined as follows 5

$$\begin{aligned} f(x) &= x^2 + x \quad &, \quad 0 \le x < 1 \\ &= 2 \quad &, \quad x = 1 \\ &= 2x^3 - x + 1 \quad &, \quad 1 < x \le 2 \end{aligned}$$

Examine the differentiability of $f$ at $x = 1$. Is $f$ continuous at $x = 1$?

(b) If $f: I \longrightarrow \mathbb{R}$ is a function differentiable at a point $c \in I$, then show that it is continuous at $c$. 3

3. (a) If $x = \sec\theta - \cos\theta,\ y = \sec^n\theta - \cos^n\theta$, show that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$. 4

(b) If $\lim\limits_{x\to 0} \frac{a\sin x - \sin 2x}{\tan^3 x}$ is finite, find the value of $a$ and the limit. 4

4. (a) If $f(x) = \sin x$, find the limiting value of $\theta$, when $h \to 0$ using the Lagrange's mean value theorem $f(a+h) = f(a) + hf'(a+\theta h)$, $0 < \theta < 1$. 4

(b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{4}{x+y+z}$. 4

5. (a) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{zx}\right)$, prove that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$. 4

(b) If $x\cos\alpha + y\sin\alpha = p$ touches the curve $\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1$, show that $(a\cos\alpha)^{\frac{m}{m-1}} + (b\sin\alpha)^{\frac{m}{m-1}} = p^{\frac{m}{m-1}}$. 4

6. (a) Find radius of curvature of the cycloid $x = a(\theta - \sin\theta)$ and $y = a(1-\cos\theta)$ at any point $\theta$. 4

(b) Find the asymptotes of the equation $(a+x)^2(b^2+x^2) = x^2y^2$. 4

7. (a) Expand $e^x$ in ascending powers of $(x-1)$. 4

(b) Verify Rolle's theorem for $f(x) = x^3 - 6x^2 + 11x - 6$ in [1, 3]. 4

8. (a) Prove that $\frac{2x}{\pi} < \sin x < x$ for $x > 0$. 4

(b) Find the greatest and the least value of $2\sin x + \sin 2x$ in the interval $\left(0, \frac{3\pi}{2}\right)$. 2+2

9. (a) Find the condition that the curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ intersect orthogonally. 4

(b) Find the points on the parabola $y^2 = 2x$ which is nearest to the point (3, 0). 4

10.(a) Find the values of $a$ and $b$ such that the function 5

$$
\begin{aligned}
f(x) &= x+\sqrt{2}a\sin x \quad , \quad 0 \le x \le \frac{\pi}{4} \\
&= 2x\cot x + b \quad , \quad \frac{\pi}{4} < x \le \frac{\pi}{2} \\
&= a\cos 2x - b\sin x \ , \quad \frac{\pi}{2} < x \le \pi
\end{aligned}
$$

is continuous for all values of $x$ in the interval $0 \le x \le \pi$ .

(b) If $u(x,\ y) = \cot^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$ , then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + \dfrac{1}{4}\sin 2u = 0$ . 3

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2021

## MTMHGEC02T/MTMGCOR02T-MATHEMATICS (GE2/DSC2)

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Test whether the equation $xdx + ydy + \dfrac{xdy - ydx}{x^2 + y^2} = 0$ is exact or not.

(b) Find an integrating factor of the differential equation $(x\log x)\dfrac{dy}{dx} + y = 2\log x$.

(c) Find particular integral of the differential equation $2x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} = \dfrac{1}{x}$.

(d) Find the transformation of the differential equation $x^2\dfrac{d^2y}{dx^2} - 5y = \log x$, using the substitution $x = e^z$.

(e) Find complementary function of the differential equation $x^2\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} = 3x$.

(f) Find the Wronskian of $y_1(x) = e^{-2x}$, $y_2(x) = xe^{-2x}$.

(g) Construct a PDE by eliminating *a* and *b* from $z = ae^{-b^2t}\cos bx$.

(h) Determine the order, degree and linearity of the following PDE:

$$\frac{\partial z}{\partial x} = \left(\frac{\partial^2 z}{\partial x^2}\right)^{5/2} + \left(\frac{\partial^2 z}{\partial y^2}\right)^{5/2}$$

(i) Classify the following PDE

$$(1+x^2)\, z_{xx} + (1+y^2)\, z_{yy} + xz_x + yz_y = 0$$

into elliptic, parabolic and hyperbolic for different values of *x* and *y*.

2. (a) Find an integrating factor of the differential equation 4

$$(2xy^4e^y + 2xy^3 + y)\, dx + (x^2y^4e^y - x^2y^2 - 3x)\, dy = 0$$

and hence solve it.

(b) Solve: $x\cos x\dfrac{dy}{dx}+y\,(x\sin x+\cos x)=1$ 4

3. (a) Find the curve for which the area of the triangle formed by $x$-axis, a tangent and the radius vector of the point of tangency is constant and equal to $a^2$. 4

(b) Using the substitution $u=\dfrac{1}{x}$ and $v=\dfrac{1}{y}$, reduce the equation $y^2(y-px)=x^4p^2$ to Clairaut's form and hence solve it. Here $p\equiv\dfrac{dy}{dx}$. 4

4. (a) Show that each of the functions $e^x, e^{4x}$ and $2e^x-3e^{4x}$ is solution of the differential equation $\dfrac{d^2y}{dx^2}-5\dfrac{dy}{dx}+4y=0,\ -\infty<x<\infty$. 2+1+1+1

Are the three independent? If not, find which two of these are independent. Write down a general solution of the equation.

(b) Find the value of $h$ so that the equation $(ax+hy+g)\,dx+(3x+by+f)\,dy=0$ becomes an exact differential equation. 3

5. (a) Solve by the method of variation of parameters: 5

$$(D^2-3D+2)y=e^x(1+e^x)^{-1},\text{ where } D\equiv\frac{d}{dx}$$

(b) Find particular integral of the differential equation 3

$$(D^2+5D+6)y=e^{-2x}\sin 2x,\quad\text{where } D\equiv\frac{d}{dx}$$

6. (a) Solve in the particular cases: 5

$$\frac{d^2x}{dt^2}-4\frac{dx}{dt}+5x=0 \text{ giving that } x=1 \text{ and } \frac{dx}{dt}=2 \text{ when } x=0$$

(b) Solve: $\dfrac{d^2y}{dx^2}=x^2\sin x$ 3

7. (a) Solve the following total differential equation: 4

$$yz\,dx+2zx\,dy-3xy\,dz=0$$

(b) Solve: $x^2\dfrac{d^2y}{dx^2}+3x\dfrac{dy}{dx}+y=x\log x$ 4

8. (a) Form a PDE by eliminating the arbitrary function $\phi$ from 4

$$lx+my+nz=\phi(x^2+y^2+z^2)$$

(b) Solve the partial differential equation by Lagrange's method $x^2p+y^2q=(x+y)z$. 4

9. (a) Find the partial differential equation of planes having equal intercepts along $x$ axis and $y$ axis. 4

(b) Find $f(y)$ such that the total differential equation $\left(\dfrac{yz+z}{x}\right)dx - zdy + f(y)\,dz = 0$ is integrable. 4

10.(a) Formulate a PDE from the relation $f\left(\dfrac{x-a}{z-c},\ \dfrac{y-b}{z-c}\right) = 0$. 3

(b) Find the Wronskian of $x$ and $|x|$ in $[-1, 1]$. 2

(c) Solve $x^2 \dfrac{d^2y}{dx^2} - 6y = 0$. 3

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2022

## MTMHGEC02T/MTMGCOR02T-MATHEMATICS (GE2/DSC2)

Time Allotted: 2 Hours Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Test whether the equation $(\sin 2x - \tan y)\,dx = x\sec^2 y\; dy$ is exact or not?

(b) Find an integrating factor of the differential equation $(2x^2 + y^2 + x)\,dx + xy\,dy = 0$.

(c) Find the differential equation of the family of parabolas $y^2 = 4ax$, where $a$ is an arbitrary constant.

(d) Verify if the following pair of functions are independent

$$e^x \quad , \quad 5e^x$$

(e) Given that $y_1(x)$, $y_2(x)$ and $y_3(x)$ are solutions of $\{D^2 + p(x)\,D + q(x)\}\,y = 0$, where $D \equiv \dfrac{d}{dx}$. Show that these solutions are linearly independent.

(f) Verify the integrability of the following differential equation:

$$yz\,dx = zx\,dy + y^2 dz$$

(g) Determine the order, degree and linearity of the following P.D.E:

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \left(\frac{\partial z}{\partial y}\right)^2 = 0$$

(h) Eliminate the arbitrary functions $\phi$ and $\psi$ from $z = \phi(x+iy) + \psi(x-iy)$, where $i^2 = -1$.

2. (a) Determine the constant $A$ of the following differential equation such that the equation is exact and solve the resulting exact equation: 4

$$\left(\frac{Ay}{x^3} + \frac{y}{x^2}\right)dx + \left(\frac{1}{x^2} - \frac{1}{x}\right)dy = 0$$

(b) Reduce the equation $\sin y \dfrac{dy}{dx} = \cos x\,(2\cos y - \sin^2 x)$ to a linear equation and hence solve it. 4

3. (a) Using the transformation $u = x^2$ and $v = y^2$ to solve the equation 4

$$xyp^2 - (x^2 + y^2 - 1)\,p + xy = 0 \quad , \quad \text{where } p = \frac{dy}{dx}$$

(b) Solve: $\frac{dy}{dx}+\frac{ax+hy+g}{hx+by+f}=0$ 4

4. (a) Solve by the method of variation of parameters: 4

$$\frac{d^2y}{dx^2}+a^2y=\cos ax$$

(b) Show that $e^x$ and $xe^x$ are linearly independent solutions of the differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$. Write the general solution of this differential equation. Find the solution that satisfies the condition $y(0)=1,\ y'(0)=4$. Is it the unique solution? 1+1+1+1

5. (a) Solve: $\{(5+2x)^2D^2-6(5+2x)D+8\}y=8(5+2x)^2$, where $D\equiv\frac{d}{dx}$. 4

(b) Solve the following equations: 4

$$\frac{dx}{dt}+4x+3y=t \quad ; \quad \frac{dy}{dt}+2x+5y=e^t$$

6. (a) Verify that the following equation is integrable, find its primitive: 5

$$zy\,dx+(x^2y-zx)\,dy+(x^2z-xy)\,dz=0$$

(b) Solve: $(4x^2y-6)\,dx+x^3\,dy=0$ 3

7. (a) Eliminate the arbitrary function $\phi$ from the relation $z=e^{my}\phi(x-y)$. 3

(b) Solve the PDE by Lagrange's method: 5

$$px(x+y)-qy(x+y)+(x-y)(2x+2y+z)=0$$

8. (a) Find the particular solution of the differential equation 4

$$(y-z)\frac{\partial z}{\partial x}+(z-x)\frac{\partial z}{\partial y}=x-y$$

which passes through the curve $xy=4,\ z=0$.

(b) Determine the points $(x,\ y)$ at which the partial differential equation 4

$$(x^2-1)\frac{\partial^2 z}{\partial x^2}+2y\frac{\partial^2 z}{\partial y\partial x}-\frac{\partial^2 z}{\partial y^2}=0$$

is hyperbolic or parabolic or elliptic.

9. (a) Solve: $(x^2+y^2+z^2)\,dx-2xy\,dy-2xz\,dz=0$ 4

(b) Solve in particular cases: 4

$$\frac{d^2y}{dx^2}+y=\sin 2x \quad ; \quad \text{when } x=0,\ y=0 \text{ and } \frac{dy}{dx}=0$$

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 3rd Semester Examination, 2021-22

## MTMHGEC03T/MTMGCOR03T-MATHEMATICS (GE3/DSC3)

### REAL ANALYSIS

Time Allotted: 2 Hours　　　　Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Find the least upper bound of the set $S=\left\{\frac{1}{p}+\frac{1}{q}\ :\ p,\ q\in\mathbb{N}\right\}$.

(b) Prove that $\mathbb{N}$ is not bounded above.

(c) Show that 0 is a cluster point of the set $S=\left\{\frac{1}{n}\ :\ n\in\mathbb{N}\right\}$.

(d) Show that $\lim\limits_{n\to\infty}\sqrt[n]{n}=1$.

(e) Examine the convergence of the series $\sum\limits_{n=1}^{\infty}\frac{(n\,!)^2}{(2n)!}$.

(f) Examine whether the sequence of functions $\{f_n\}$ converges uniformly on $\mathbb{R}$, where for all $n\in\mathbb{N}$,

$$f_n(x)=\frac{x}{n}\ ,\ \text{for all } x\in\mathbb{R}.$$

(g) Show that the series $\sum\limits_{n=1}^{\infty}\frac{1}{n^3+n^4x^2}$ is uniformly convergent on $\mathbb{R}$.

(h) Find the radius of convergence of the power series $\sum\limits_{n=1}^{\infty}\frac{(-1)^n n!}{n^n}x^n$.

(i) Show that the sequence $\left\{\frac{1}{n}\right\}$ is a Cauchy sequence.

2. (a) If $S$ is a non empty subset of $\mathbb{R}$ and also bounded below then prove that $S$ has an infimum. 3

(b) Show that the subset $S=\{x\in\mathbb{Q}\ :\ x>0\ ,\ x^2<2\}$ is a non empty subset of $\mathbb{Q}$, bounded below; but $\inf S$ does not belong to $\mathbb{Q}$. 5

3. (a) Show that 0 is a limit point of the set $\{x\ :\ 0<x<1\}$. 2

(b) Find all limit points of the set of all rational numbers $\mathbb{Q}$. 3

(c) Prove that $\mathbb{Z}$ is not bounded below. 3

4. (a) Prove that the set of all open intervals having rational end points is enumerable. 4

(b) Show that the sequence $\left\{\dfrac{n^2+2022}{n^2}\right\}$ converges to 1. 4

5. (a) Show that the sequence $\{x_n\}$ is monotone increasing, where 4

$$x_n = 1 + \frac{1}{2} + \ldots\ldots + \frac{1}{n} \quad \text{for all } n \in \mathbb{N}$$

Hence show that the sequence $\{x_n\}$ is not convergent.

(b) Apply Cauchy's criterion for convergence to show that the sequence $\{x_n\}$ is convergent, where 4

$$x_n = 1 + \frac{1}{2!} + \ldots\ldots + \frac{1}{n!} \quad \forall\ n \in \mathbb{N}$$

6. (a) Let $x \in \mathbb{R}$. Show that the series $\sum_{n=1}^{\infty} \frac{x^n}{n^n}$ converges absolutely. 4

(b) Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+1)}$. 4

7. (a) Discuss the convergence of the series $\sum 1/n^p$ , $p > 0$. 4

(b) Let $f_n(x) = x^n$ , $x \in [0, 1]$. Show that the sequence of function $\{f_n\}$ is not uniformly convergent on [0, 1]. 4

8. (a) Let $f_n(x) = nxe^{-nx^2}$ , $x \in [0, 1]$, $n \in \mathbb{N}$. Show that the sequence $\{f_n\}$ is not uniformly convergent on [0, 1]. 4

(b) Prove that the series $\sum \frac{x}{n + n^2 x^2}$ is uniformly convergent for all real $x$. 4

9. (a) Show that the series $x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \ldots\ldots$ is not uniformly convergent on [0, 1]. 4

(b) Let $f : \mathbb{R} \to \mathbb{R}$ be uniformly continuous on $\mathbb{R}$. For each $n \in \mathbb{N}$, let $f_n(x) = f\left(x + \frac{1}{n}\right)$ , $x \in \mathbb{R}$. Prove that the sequence $\{f_n\}$ is uniformly convergent on $\mathbb{R}$. 4

10.(a) If $\{u_n\}$ be a sequence of real numbers and $\sum u_n^2$ is convergent prove that $\sum \frac{u_n}{n}$ is absolutely convergent. 4

(b) If $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences then prove that, 4

(i) $\{x_n + y_n\}$ is a Cauchy sequence

(ii) $\{x_n y_n\}$ is a Cauchy sequence.

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 4th Semester Examination, 2022

## MTMHGEC04T/MTMGCOR04T-MATHEMATICS (GE4/DSC4)

Time Allotted: 2 Hours Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) In $\mathbb{Z}_{14}$, find the smallest positive integer $n$ such that $n[6]=[0]$. 2

(b) Let $(G,*)$ be a group. If every element of $G$ has its own inverse then prove that $G$ is commutative. 2

(c) Let $H$ be a subgroup of a group $G$. Show that for all $a\in G$, $aH=H$ if and only if $a\in H$. 2

(d) Check whether the relation $\rho$ defined by $x\rho y$ if and only if $|x|=|y|$, is an equivalence relation or not on the set of integers $\mathbb{Z}$. Justify your answer. 2

(e) Show that the alternative group $A_3$ is a normal subgroup of $S_3$. 2

(f) Show that every cyclic group is abelien. 2

(g) Show that the ring of matrices $\left\{\begin{pmatrix} 2a & 0 \\ 0 & 2b \end{pmatrix} : a, b\in \mathbb{Z}\right\}$ contains divisors of zero and does not contain the unity. 2

(h) Let $A$ and $B$ be two ideals of a ring $R$. Is $A\cup B$ an ideal of $R$? Justify. 2

2. (a) A relation $\rho$ on the set $\mathbb{N}$ is given by $\rho=\{(a,b)\in \mathbb{N}\times\mathbb{N} : a \text{ is a divisor of } b\}$. Examine if $\rho$ is (i) reflexive, (ii) symmetric, (iii) transitive. 4

(b) If $G$ is a group such that $(ab)^2=a^2b^2$ for all $a,b\in G$; then show that $G$ is commutative. 4

3. (a) Let $A=\{1,2,3\}$. List all one-one functions from $A$ onto $A$. 4

(b) Let $G$ be a commutative group. Show that the set $H$ of all elements of finite order is a subgroup of $G$. 4

4. (a) Let $H$ be a subgroup of a group $G$. Show that the relation $\rho$ defined on $G$ by "$a\rho b$ if and only if $a^{-1}b\in H$" for $a,b\in G$ is an equivalence relation. 4

(b) Prove that the order of every subgroup of a finite group $G$ is a divisor of the order of $G$. 4

5. (a) Prove that every group of order less than 6 is commutative. 4

(b) Let $(G, \circ)$ be a cyclic group generated by $a$. Then prove that $a^{-1}$ is also a generator. 4

6. (a) Show that the intersection of two normal subgroups of a group $G$ is normal in $G$. 4

(b) Show that if $H$ be a subgroup of a commutative group $G$ then the quotient group $G/H$ is commutative. Is the converse true? Justify. 4

7. (a) Prove that an infinite cyclic group has only two generators. 4

(b) In the rings $\mathbb{Z}_8$ and $\mathbb{Z}_6$, find the following elements: 2+2

(i) the units and (ii) the zero divisors.

8. (a) Find all ideals of $\mathbb{Z}$. 4

(b) Let $R$ be a commutative ring with 1. Then prove that $R$ is a field if and only if $R$ has no non-zero proper ideals. 4

9. (a) (i) Let $S$ be a set with $n$ elements. How many binary operations can be defined on $S$? Justify. 2+2

(ii) Let $A$ and $B$ be two sets with $|A| = 5$ and $|B| = 2$. How many surjective functions defined from $A$ onto $B$? Justify.

(b) Let $G = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a\,(\neq 0) \in \mathbb{R} \right\}$. Show that $G$ forms a group w.r.t. matrix multiplication. 4

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 6th Semester Examination, 2022

## MTMGDSE03T-MATHEMATICS (DSE2)

### NUMERICAL METHODS

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) Write down the relations of Central difference operator, $\delta$ and Average operator, $\mu$ with the shift operator $E$.

(b) Obtain two consecutive integers between which there is a root of $x^3+x+5=0$.

(c) Write down the number $\frac{2}{3}$ correct upto 5 significant figures and find relative error.

(d) Why is the Newton-Raphson method for computing a simple root of an equation $f(x)=0$ called method of tangents?

(e) Construct a linear interpolation for $f(x)$ with $f(1)=3$ and $f(2)=-5$.

(f) Show that $\Delta\log f(x)=\log[1+\Delta f(x)/f(x)]$

(g) Find the value of $f'(0.2)$ using the table of values of $f(x)$

| $x$ | 0.2 | 0.4 | 0.6 |
|---|---|---|---|
| $f(x)$ | 1.6596 | 1.6698 | 1.6804 |

(h) Using trapezoidal rule compute $\int_0^2 f(x)\,dx$. Given

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 1.6 | 3.8 | 8.2 |

2. (a) Find a real root of the equation $3x-\cos x-1=0$ correct to two significant figures by using Newton Raphson method. 4

(b) Discuss method of bisection for computing a real root of an equation $f(x)=0$. 4

3. (a) Find Lagrange's interpolation polynomial for the function $f(x)=\sin\pi x$, when $x_0=0$, $x_1=\frac{1}{6}$, $x_2=\frac{1}{2}$. Also compute the value of $\sin\frac{\pi}{3}$ and estimate the error. 3+1+1

(b) Find $f(5)$, given that $f(0)=-2$, $f(1)=4$, $f(2)=6$, $f(3)=10$ and third difference being constant. 3

4. (a) Solve the equation 6

$$2x+3y+z=9$$
$$x+2y+3z=6$$
$$3x+y+2z=8$$

by the method of matrix factorization

(b) Round off the number 40.3586 and 0.0056812 to four significant digits. 2

5. (a) Find the missing terms in the following table: 4

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $y$ | 1 | 5 | – | 121 | – | 781 |

(b) Use of Stirling interpolation formula prove that 4

$$\frac{d}{dx}f(x)=\frac{2}{3}[f(x+1)-f(x-1)]-\frac{1}{12}[f(x+2)-f(x-2)],$$

considering the differences upto third order.

6. (a) Compute $f(0.5)$ from the following table 4

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f(x)$ | 1 | 2 | 11 | 34 |

(b) Show that $n$th order difference of a polynomial of degree $n$ are constant. Does the converse of the result true? 4

7. (a) Evaluate numerically the integration $\int_0^1 \frac{1}{1+x}dx$, by Simpson's $\frac{1}{3}$rd rule taking 6 equal subintervals. 4

(b) If $f(x)$ is a polynomial of degree 2, prove that 4

$$\int_0^1 f(x)dx=[5f(0)+8f(1)-f(2)]\Big/12.$$

8. (a) Compute by the method of fixed point iteration method the positive root of the equation $x^2-x-0.1=0$ correct upto three significant figures. 4

(b) Find the real root of the equation $x^3-x-1=0$ by Regula-Falsi method correct upto two significant figures. 4

9. (a) Use Euler's method with $h=0.2$ to find the solution of $\frac{dy}{dx}=2x+y,\ y(0)=1$ at $x=0.4$. 5

(b) Find the location of the positive roots of $x^3-9x+1=0$, and evaluate the smallest one by bisection method correct to two decimal places. 3

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 6th Semester Examination, 2022

## MTMGDSE04T-MATHEMATICS (DSE2)

### LINEAR PROGRAMMING

Time Allotted: 2 Hours  Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**GROUP-A**

**Full Marks-10**

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Is the set $X=\{(x,y): x^2+y^2\le 4\}$ is convex? Justify your answer.

(b) In the following equations find the basic solution with $x_3$ as the non-basic variable

$$x_1+4x_2-x_3=3$$
$$5x_1+2x_2+3x_3=4$$

(c) Find a basic feasible solution of the equations $x_1+x_2+x_3=8$, $3x_1+2x_2=18$

(d) Find the extreme points, if any, of the set $S=\{(x,y): 2x+3y=6\}$

(e) Draw the convex hull of the points $(0, 0)$, $(0, 1)$, $(1, 2)$, $(1, 1)$, $(4, 0)$.

(f) Write down the dual of the following L.P.P.:

Maximize $Z=3x_1+5x_2$

Subject to $x_1+2x_2\le 5$

$x_1-x_2=7$

$x_1,\ x_2\ge 0$

(g) Determine the position of the point (–1, 2, 5, 3) relative to the hyperplane

$$4x_1+6x_2+x_3-3x_4=4$$

(h) Find the number of basic feasible solutions of the following L.P.P.:

Maximize $Z=2x_1+3x_2$

Subject to $x_1+x_2\ge 2$

$x_1-x_2\le 1$

$x_1,\ x_2\ge 0$

(i) What is the criterion for no feasible solution in two-phase method?

## GROUP-B

## Full Marks-40

**Answer any *five* questions from the following** $8\times5=40$

2. (a) Solve the following L.P.P using graphical method 4

Maximize $Z=2x_1+x_2$

Subject to $4x_1+3x_2\le 12$

$4x_1+x_2\le 8$

$4x_1-x_2\le 8$

$x_1,\ x_2\ge 0$

(b) Food $X$ contains 6 units of vitamin A and 7 units of vitamin B per gram and costs 12 p./gm. Food $Y$ contains 8 units of vitamin A and 12 units of vitamin B per gram and costs 20 p./gm. The daily requirements of vitamin A and B are at least 100 units and 120 units respectively. Formulate the above as an L.P.P. to minimize the cost. 4

3. (a) Use Simplex method to solve the L.P.P. 4

Maximize $Z=x_1+2x_2+4x_3$

Subject to $3x_1+5x_2+2x_3\le 6$

$4x_1+4x_3\le 7$

$2x_1+4x_2-x_3\le 10$

$x_1,\ x_2,\ x_3\ge 0$

(b) Show that the vectors $(1,-2,0),(3,1,2),(5,-1,4)$ form a basis in $E^3$. 4

4. (a) Prove that the set of all convex combinations of a finite number of points is a convex set. 5

(b) Find a supporting hyperplane of the convex set 3

$S=\{(x,y): x+2y\le 4, 3x+y\le 6, x\ge 0, y\ge 0\}$

5. (a) $x_1=1,\ x_2=1,\ x_3=1,\ x_4=0$ is a feasible solution of the system of equations 5

$x_1+2x_2+4x_3+x_4=7$

$2x_1-x_2+3x_3-2x_4=4$

Reduce the feasible solution to two different basic feasible solutions.

(b) Prove that a hyperplane is a convex set. 3

6. (a) Obtain a basic feasible solution of the following L.P.P. from the feasible solution (2, 3, 1) 4

Maximize $Z=x_1+2x_2+4x_3$

Subject to $2x_1+x_2+4x_3=11$

$3x_1+x_2+5x_3=14$

$x_1,\ x_2,\ x_3\ge 0$

(b) Prove that the intersection of two convex sets is also a convex set. 4

7. (a) Solve by Charnes Big M-method the following L.P.P. 6

Maximize $Z = 4x_1 + x_2$

Subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$x_1,\ x_2 \geq 0$

(b) Discuss whether the set of points $(0, 0), (0, 1), (1, 0), (1, 1)$ on the $xy$-plane is a convex set or not. 2

8. (a) Prove that dual of a dual is a primal. 4

(b) Obtain the dual problem of the following L.P.P. 4

Maximize $Z = -x_1 + 3x_2$

Subject to $2x_1 + x_2 \leq 1$

$3x_1 + 4x_2 \leq 5$

$x_1 + 6x_2 \leq 9$

$x_1,\ x_2,\ x_3 \geq 0$

9. (a) Find the points which generate the convex polyhedron 3

$$S = \{(x_1, x_2) \in E^2 : x_1 + 2x_2 \leq 4,\ x_1 - 2x_2 \leq 2,\ x_1 \geq 0,\ x_2 \geq 0\}$$

(b) Use two-phase method to solve the following L.P.P. 5

Maximize $Z = 3x_1 + 5x_2$

Subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$5x_1 + 6x_2 \leq 60$

$x_1, x_2 \geq 0$

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