WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2022-23

MTMACOR01T-MATHEMATICS (CC1)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Evaluate $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$.

(b) If (2, 5/2) is known to be a point of inflection of the curve $3x^2y+\alpha x+\beta y=0$, then find the value of α and β.

(c) Find the interval where the curve $y=e^x(\cos x+\sin x)$ is concave upwards or downwards for $0<x<2\pi$.

(d) Write the equation $4xy=1$ in terms of a rotated rectangular $x'y'$-system if the axes are turned through an angle $\tan^{-1}2$.

(e) Show that the abscissa of the points of inflexion on the curve $y^2=f(x)$ satisfy the equation $\{f'(x)\}^2=2f(x)f''(x)$.

(f) Find the equation of the generating lines of the hyperboloid $yz+2zx+3xy+6=0$ which pass through the point $(-1, 0, 3)$.

(g) Find the equation of the sphere which passes through the circle $x^2+y^2+z^2=4$, $z=0$ and is cut by the plane $x+2y+2z=0$ in a circle of radius 3 units.

(h) Find the value of a and b for which the differential equation $(3a^2x^2+by\cos x)\,dx+(2\sin x-4ay^3)\,dy=0$ is exact.

(i) Show that the equation $\frac{dy}{dx}=2y^{1/2}$, $y(0)=0$ has no unique solution.

2. (a) If $y^{\frac{1}{m}}+y^{-\frac{1}{m}}=2x$, prove that 4

$$(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$$

(b) Prove that the points of inflexion of the curve $y^2(x-a)=x^2(x+a)$ subtend an angle $\frac{\pi}{3}$ at the origin. 4

3. (a) Show that envelope of the lines drawn at right angles to the radii vectors of the cardioid $r=a(1+\cos\theta)$ through their extremities is given by $r=2a\cos\theta$. 4

(b) Find the asymptotes of the curve $r=\frac{a}{\frac{1}{2}-\cos\theta}$. 4

4. (a) Trace the curve given by the equation 4

$$y^2=x^2\left(\frac{a+x}{a-x}\right).$$

(b) Show that the equation $4x^2-4xy+y^2+2x-26y+9=0$ represents a parabola whose latus rectum is $2\sqrt{5}$ units. 4

5. (a) Prove that length of the arc of the parabola $y^2=4ax$ which is intercepted between the points of intersection of the parabola and the straight line $3y=8x$ is $a\left(\log 2+\frac{15}{16}\right)$. 4

(b) A sphere of constant radius '*d*' through the origin and intersects the co-ordinate axes in P, Q, R. Prove that the centroid of the triangle PQR lies on the sphere $9(x^2+y^2+z^2)=4d^2$. 4

6. (a) Find the equation of the sphere which passes through the origin and touches the sphere $x^2+y^2+z^2=56$ at the point $(2,\,-4,\,6)$. 4

(b) Find the equation of the cylinder whose generators are parallel to the straight line $2x=y=3z$ and which passes through the circle $x^2+z^2=6$, $y=0$. 4

7. (a) Through a variable generator $x-y=\lambda$, $x+y=\frac{2z}{\lambda}$ of the paraboloid $x^2-y^2=2z$ a plane is drawn, making an angle $\frac{\pi}{4}$ with the plane $x=y$. Find the locus of the point at which it touches the paraboloid. 4

(b) The curve that an idealised hanging chain or cable assumes when supported at its ends and acted on solely by its own weight is called a catenary. The equation of this curve is 4

$$y=a\cosh\left(\frac{x}{a}\right)=\frac{a}{2}(e^{x/a}+e^{-x/a})$$

Find the arc length of the curve between the points where it is cut by $y=2a$.

8. (a) Determine the surface area of the solid obtained by rotating $y=\sqrt{9-x^2}$, $|x|\le 2$ about the *x*-axis. 4

(b) Show that the following first order ode is exact and hence solve it. 4

$$\left(\frac{1+8xy^{2/3}}{x^{2/3}y^{1/3}}\right)dx+\left(\frac{2x^{4/3}y^{2/3}-x^{1/3}}{y^{4/3}}\right)dy=0.$$

9. (a) Find suitable integrating factor of the following ode and hence solve it. 4

$$(6+12x^2y^2)+\left(7x^3y+\frac{x}{y}\right)\frac{dy}{dx}=0.$$

(b) Find singular solution of $9\left(\frac{dy}{dx}\right)^2(2-y)^2=4(3-y)$. 2

(c) Solve: $(4x^2y-6)\,dx+x^3dy=0$. 2

10.(a) Determine the constants *a*, *b*, *c* such that 4

$$\lim_{x\to 0}\frac{x(a+b\cos x)+c\sin x}{x^5}=\frac{1}{60}$$

(b) Show that the differential equation of the circles through the intersection of the circle $x^2+y^2=1$ and the line $x-y=0$ is given by 4

$$(x^2-2xy-y^2+1)\,dx+(x^2+2xy-y^2-1)\,dy=0.$$

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2022-23

MTMACOR02T-MATHEMATICS (CC2)

ALGEBRA

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) If a, b, c, d are positive real numbers, not all equal, prove that

$$(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)>16$$

(b) Prove that $\sqrt[n]{i}+\sqrt[n]{-i}=2\cos\frac{\pi}{2n}$, where n is a positive integer greater than 1 and $\sqrt[n]{z}$ is the principal nth root of z.

(c) Apply Descartes' rule of sign to find the least number of non real roots of the equation $x^{10}-x^3=0$.

(d) If a and b are integers s.t. g.c.d. $(a, b)=1$ then prove that g.c.d $(a+b, a\cdot b)=1$.

(e) Show that the product of any four consecutive integers is divisible by 24.

(f) Give an example of a surjective mapping $f: S\to S$ which is not injective, where S is an infinite set.

(g) Find a relation on the set of positive integers which is transitive but neither reflexive nor symmetric.

(h) Solve the equation $2x^3-x^2-18x+9=0$ if two of its roots are equal in magnitude but opposite in sign.

(i) Give an example of a 3×3 matrix whose eigenvalues are 1, 2 and 3.

2. (a) If $3s=a+b+c+d$, where a, b, c, d and $s-a$, $s-b$, $s-c$, $s-d$ are all positive, prove that 4

$$abcd>81(s-a)(s-b)(s-c)(s-d),$$

unless $a=b=c=d$.

(b) If a, b, c are positive real numbers s.t. $a+b+c=1$, then prove that 4

$$\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2+\left(c+\frac{1}{c}\right)^2\geq 33\frac{1}{3}$$

3. (a) If α be a special root of the equation $x^{12}-1=0$, prove that 4

$$(\alpha+\alpha^{11})(\alpha^5+\alpha^7)=-3$$

(b) Solve the equation $x^4-6x^2-16x-15=0$ by Ferrari's method. 4

4. (a) Show that the principal value of the ratio of $(1+i)^{1-i}$ and $(1-i)^{1+i}$ is 4

$$\sin(\log 2)+i\cos(\log 2).$$

(b) If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ then show that 4

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=\frac{3}{2}=\sin^2\alpha+\sin^2\beta+\sin^2\gamma.$$

5. (a) Prove that for all $n\in\mathbb{N}$, $(2+\sqrt{3})^n+(2-\sqrt{3})^n$ is an even integer. 4

(b) Prove that $13^{73}+14^3\equiv 2 \pmod{11}$. 4

6. (a) Examine whether the relation ρ is an equivalence relation on the set $\mathbb{Z}$ of all integers, where 4

$$\rho=\{(m,n)\in\mathbb{Z}\times\mathbb{Z}:|m-n|\le 3\}.$$

(b) Suppose that $f:A\to B$ is a function. Show that f is $1-1$ if and only if there exists an onto function $g:B\to A$ satisfying $g(f(a))=a, \forall a\in A$. 2+2

7. (a) Prove that the eigenvalues of a real symmetric matrix are all real. 4

(b) Compute the inverse of the following matrix by row transformations: 4

$$A=\begin{pmatrix}3 & 1 & 1\\ 4 & 2 & -1\\ 7 & 3 & 1\end{pmatrix}.$$

8. (a) Determine the conditions for which the system of equation has 4

(i) only one solution (ii) no solution (iii) many solutions

$$x+y+z=1$$
$$x+2y-z=b$$
$$5x+7y+az=b^2$$

(b) Reduce the matrix 4

$$A=\begin{pmatrix}1 & 2 & -2\\ 2 & 1 & 2\\ -2 & 2 & 1\end{pmatrix}$$

to a row-reduced Echelon form and find its rank.

9. (a) Show that the eigen vectors corresponding to distinct eigenvalues of an $n\times n$ matrix A are linearly independent. 4

(b) Find the eigenvalues and the corresponding eigen vectors of the following matrix 4

$$\begin{bmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{bmatrix}$$

10.(a) Show that zero is a characteristic root of a matrix A if and only if A is singular. 3

(b) Verify Cayley-Hamilton theorem for the matrix 5

$$A=\begin{pmatrix}1 & 2 & 1\\ 1 & -1 & 1\\ 2 & 3 & -1\end{pmatrix}$$

Express A^{-1} as a polynomial in A and then compute A^{-1}.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2023

MTMACOR03T-MATHEMATICS (CC3)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) State Supremum property and Archimedean property of R, the set of all real numbers. 1+1

(b) Is the set $\{x \in R: \sin x \neq 0\}$ open in R? Justify your answer.

(c) Verify Bolzano-Weierstrass theorem for the set $\left\{\frac{n}{n+1}: n \in \mathbb{N}\right\}$.

(d) Prove that the sequence $\{x_n\}$ where $x_n = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \cdots\cdots + \frac{1}{(2n-1)(2n+1)}$ is bounded.

(e) If $A = [-1, 4)$ and $B = (2, 5]$, is $A \cup B$ compact? Give reasons.

(f) Show that the sequence $\{x_n\}$ is a null sequence where $x_n = \frac{n!}{n^n}$.

(g) Use comparison test to examine the convergence of the series:

$$\frac{1}{1\cdot 2^2} + \frac{1}{2\cdot 3^2} + \frac{1}{3\cdot 4^2} + \cdots\cdots$$

(h) Test the convergence of the series:

$$1 - \frac{2^2}{2!} + \frac{3^3}{3!} - \frac{4^4}{4!} + \cdots\cdots$$

2. (a) Let A and B be two non-empty bounded sets of real numbers. Let $C = \{x + y : x \in A, y \in B\}$. Show that $\sup C = \sup A + \sup B$. 3

(b) If S be a subset of R, then prove that interior of S is an open set. 2

(c) Prove that the set $\mathbb{Q}$ of rational numbers is enumerable. 3

3. (a) If $S = \{(-1)^m + \frac{1}{n};\ m \in N,\ n \in N\}$, then find the derived set of S. Is S a closed set? Justify your answer. 3

(b) If G is an open set in R then prove that $R - G$ is closed. 2

(c) Let $S = \bigcup_{n=1}^{\infty} I_n$, where $I_n = \left\{x \in R : \frac{1}{2^n} \leq x \leq 1\right\}$. Is the set S closed? Justify your answer. 3

4. (a) Prove that every compact subset of R is closed and bounded. 5
(b) Give an example of a set which is closed, but not compact. Give reasons. 1
(c) Prove that the intersection of two compact sets in R is compact. 2

5. (a) State and prove Sandwich theorem for convergence of a sequence and use it to prove that $\lim_{n\to\infty}(2^n+3^n)^{1/n}=3$. 1+2+2
(b) If $u_1>0$ and $u_{n+1}=\frac{1}{2}\left(u_n+\frac{9}{u_n}\right)$, $\forall\ n\geq 1$, then show that $\{u_n\}$ is monotonically decreasing and bounded below. Is it convergent? 3

6. (a) If a sequence $\{u_n\}$ converges to l, then prove that every subsequence of $\{u_n\}$ converges to l. 2
(b) If the n-th term of the sequence $\{u_n\}$ is given by $u_n=(-1)^n+\sin\frac{n\pi}{4}$, $n=1, 2, 3, \cdots\cdots$, then find two subsequences of $\{u_n\}$, one converging to the upper limit and the other converging to the lower limit. Is the sequence convergent? Give reasons. 3
(c) Show that $\lim_{n\to\infty}\frac{1\cdot 3\cdot 5\cdot\cdots\cdots(2n-1)}{2\cdot 4\cdot 6\cdot\cdots\cdots 2n}=0$. 3

7. (a) Prove that every convergent sequence is bounded. Is the converse true? Give reasons. 2+1
(b) Using definition of Cauchy sequence, show that the sequence $\{\frac{1}{n}\}$ is a Cauchy sequence. 2
(c) Prove or disprove: A monotone sequence of real numbers having a convergent subsequence is convergent. 3

8. (a) State and prove Leibnitz test for convergence of an alternating series. 1+3
(b) Use this to test the convergence of the series $\sum_{n=2}^{\infty}\frac{(-1)^{n+1}}{n\log n}$. 2
(c) Define conditionally convergent series with example. 1+1

9. (a) Use Cauchy's integral test to show that $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges for $p>1$ and diverges for $p\leq 1$. 3
(b) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}.(n!)^2.7^n}{(2n)!}$. 3
(c) If $\sum_{n=1}^{\infty}a_n$ is a convergent series of positive real numbers, will the series $\sum_{n=1}^{\infty}a_{2n}$ be convergent? Justify your answer. 2

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2023

MTMACOR04T-MATHEMATICS (CC4)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Explain, with the help of uniqueness and existence theorem, that the differential equation

$$\frac{dy}{dx}=\frac{y}{x}$$

has infinite number of solutions passing through the point (0, 0).

(b) Show that $e^x \sin x$ and $e^x \cos x$ are linearly independent solutions of the differential equation

$$\frac{d^2y}{dx^2}-2\cdot\frac{dy}{dx}+2y=0$$

(c) Solve $(D^2-4D)y=x^2$, $\left(D\equiv\frac{d}{dx}\right)$ by using the method of undetermined coefficients.

(d) Find the particular integral of the differential equation

$$(D^2-1)y=e^{-x},\ \left(D\equiv\frac{d}{dx}\right)$$

(e) Locate and classify the singular points of the equation

$$x^3(x-2)\frac{d^2y}{dx^2}-(x-2)\frac{dy}{dx}+3xy=0$$

(f) Find the magnitude of the volume of the parallelopiped having the vectors $\vec{a}=-3\hat{i}+7\hat{j}+5\hat{k}$, $\vec{b}=5\hat{i}+7\hat{j}-3\hat{k}$ and $\vec{c}=7\hat{i}-5\hat{j}-3\hat{k}$ as the concurrent edges.

(g) If $\vec{F}=y\hat{i}-xz\hat{j}+x^2\hat{k}$ and C be the curve $x=t$, $y=2t^2$, $z=t^3$ from $t=0$ to $t=1$, then evaluate the integral $\int_C \vec{F}\times d\vec{r}$.

(h) A particle moves so that its position vector is given by $\vec{r}=\cos\omega t\,\hat{i}+\sin\omega t\,\hat{j}$, where ω is a constant. Show that the acceleration $\vec{a}$ is directed towards the origin and has magnitude proportional to the distance from the origin.

2. (a) If $\vec{a}\times\vec{b}=\vec{c}$, $\vec{b}\times\vec{c}=\vec{a}$ and $\vec{c}\times\vec{a}=\vec{b}$, then show that $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular. 4

(b) Show that in general $(\vec{a}\times\vec{b})\times\vec{c}\neq\vec{a}\times(\vec{b}\times\vec{c})$; but if the equality holds, then either $\vec{b}$ is parallel to $(\vec{a}\times\vec{c})$ or $\vec{a}$ and $\vec{c}$ are collinear. 4

3. (a) Integrate the function $\vec{F}=x^2\hat{i}-xy\hat{j}$ from (0, 0) to (1, 1) along the parabola $y^2=x$. 4

(b) Prove that the necessary and sufficient condition for the vector function $\vec{a}(t)$ to have constant magnitude is $\vec{a}\times\frac{d\vec{a}}{dt}=\vec{0}$. 4

4. (a) If $\vec{r}(t)=2\hat{i}-\hat{j}+2\hat{k}$ when $t=2$ and $\vec{r}(t)=4\hat{i}-2\hat{j}+3\hat{k}$ when $t=3$, then show that 4

$$\int_2^3\left(\vec{r}\cdot\frac{d\vec{r}}{dt}\right)dt=10$$

(b) Find the unit tangent, the curvature, the principal normal, the binormal and the torsion for the space curve 4

$$x=t-\frac{t^3}{3}\ ,\quad y=t^2\ ,\quad z=t+\frac{t^3}{3}$$

5. (a) Solve $x^2\frac{d^2y}{dx^2}-3x\cdot\frac{dy}{dx}+y=\frac{\log_e x\sin\log_e x+1}{x}$. 4

(b) If y_1 and y_2 be two independent solutions of the linear homogeneous equation 4

$$\frac{d^2y}{dx^2}+P\cdot\frac{dy}{dx}+Q\cdot y=0$$

then show that the Wronskian $W(y_1, y_2)$ is given by

$$W(y_1, y_2)=A\cdot e^{-\int P\cdot dx},\ \text{where } A \text{ is a constant.}$$

6. (a) Solve the equation 4

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}=x+e^x\sin x$$

by the method of undetermined coefficients.

(b) Solve 4

$$\frac{d^2y}{dx^2}+2\cdot\frac{dy}{dx}+y=\frac{e^{-x}}{x^2}$$

by the method of variation of parameters.

7. (a) Solve 4

$$\frac{d^2x}{dt^2}+\frac{dy}{dt}+x+y=t \quad , \quad \frac{dy}{dt}+2x+y=0$$

given that $x=y=0$ at $t=0$.

(b) Solve: $\frac{d^2y}{dx^2}+4y=4\tan 2x$ 4

8. (a) Solve the equation $\frac{d^2y}{dx^2}+(x-1)^2\frac{dy}{dx}-4(x-1)y=0$ in series about the point $x=1$. 5

(b) Show that the point of infinity is a regular singular point of the equation 3

$$x^2\cdot\frac{d^2y}{dx^2}+(3x-1)\frac{dy}{dx}+3y=0$$

9. (a) Solve: $(D^3-1)y=\cos^2\frac{x}{2}$ 4

(b) Solve $x^2\cdot\frac{d^2y}{dx^2}+x\cdot\frac{dy}{dx}-y=0$, given that $x+\frac{1}{x}$ is one integral. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 3rd Semester Examination, 2022-23

MTMACOR05T-MATHEMATICS (CC5)

THEORY OF REAL FUNCTIONS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5=10$

(a) Prove that $\lim\limits_{x\to 0} x^{3/2}=0$.

(b) Show that $\lim\limits_{x\to 0}\cos\frac{1}{x}$ does not exist.

(c) Determine the value of a so that

$$f(x)=\begin{cases} x+1 & ; \quad x\le 1 \\ 3-ax^2 & ; \quad x>1 \end{cases}$$

is continuous at $x=1$.

(d) Give an example of two functions $f, g: I\to\mathbb{R}$, where I is an interval in $\mathbb{R}$, which are not continuous at a point $c\in I$, but $f+g: I\to\mathbb{R}$ is continuous at c.

(e) Show that the function $f:[1, 2]\to\mathbb{R}$ defined by

$$f(x)=\begin{cases} x; & x\in[1, 2]\cap\mathbb{Q} \\ -x; & x\in[1, 2]-\mathbb{Q} \end{cases}$$

is discontinuous at every point of $[1, 2]$.

(f) Examine the differentiability of $f(x)$ at $x=0$ and $x=1$ where

$$f(x)=\begin{cases} 1-x^2, & -1\le x<0 \\ x^2+x+1, & 0\le x<1 \\ x^3+2, & 1\le x\le 2 \end{cases}$$

(g) Examine validity of Rolle's theorem for the function

$$f(x)=\sin x\cos x, \qquad x\in\left[0, \frac{\pi}{2}\right]$$

Also, verify the conclusion of Rolle's theorem for this function, if possible.

(h) Examine the validity of the hypothesis and conclusion of Lagrange's mean value theorem for the following function:

$$f(x)=x(x-1)(x-2) \quad x\in\left[0,\frac{1}{2}\right]$$

(i) Find the maximum value of the function

$$y=1+2\sin x+3\cos^2 x, \quad 0\le x\le\frac{\pi}{2}.$$

2. (a) Let $f:D\to\mathbb{R}$, where $D\subseteq\mathbb{R}$ and let $\lim_{x\to a} f(x)=l$. Show that there is a neighborhood N of a so that f is bounded on $(N-\{a\})\cap D$. 4

(b) Show that 4

$$\lim_{x\to\infty}\frac{x+[x]}{x^2}=0$$

where $[x]$ denotes the integral part of x for any $x\in\mathbb{R}$.

3. (a) Let $f:I\to\mathbb{R}$ and $g:J\to\mathbb{R}$ be such that Image $f\subseteq J$, f is continuous at $a\in I$ and g is continuous at $f(a)\in J$. Show that the composition $g\circ f:I\to\mathbb{R}$ is continuous at a. 4

(b) Let $f:I\to\mathbb{R}$ be a function continuous at $c\in I$, where I is an interval in $\mathbb{R}$. Let f take both positive and negative values in each neighborhood of c. Show that $f(c)=0$. 4

4. (a) Let $f:D\to R$ $(D\subset R)$ be a function and c be a limit point of D. Let $l\in R$, then prove that $\lim_{x\to c} f(x)=l$ if and only if for every sequence $\{x_n\}$ in $D-\{c\}$ converging to c, the sequence $\{f(x_n)\}$ converges to l. 2+3

(b) Using the above theorem prove that $\lim_{x\to 0}\cos\frac{1}{x}$ does not exist. 3

5. (a) Prove that every continuous function f on a closed and bounded interval $[a, b]$ is bounded and there exists a point $c\in[a, b]$ such that 3+2

$$f(c)=\sup_{x\in[a,\,b]} f(x)$$

(b) Let $I=[a, b]$ be a closed and bounded interval and $f:[a, b]\to R$ be continuous on I, then prove that $f(I)=\{f(x):x\in I\}$ is a closed and bounded interval. 3

6. (a) Is a function $f:I\to R$ which is uniformly continuous on I, continuous on I? Give reason. 3+1

Under what condition a continuous function $f:I\to R$ will be uniformly continuous?

(b) If $f: D \rightarrow R$ $(D \subset R)$ be uniformly continuous on D and $\{x_n\}$ be a Cauchy sequence in D, then prove that $\{f(x_n)\}$ is a Cauchy sequence in R. Using this, prove that $f(x)=\frac{1}{x}$, is not uniformly continuous on $(0, 1)$. 2+2

7. If $f: I \rightarrow R$ is differentiable at $c \in I$, then prove that f is increasing at $x=c$ if $f'(c)>0$. 2+2+4

Is the condition necessary for a function to be increasing at a point? Give reason.

Use this result to prove that

$$\frac{x}{1+x} < \log(1+x) < x \quad \text{for all } x > 0.$$

8. (a) Let a function f be derivable in some closed and bounded interval $[a, b]$ and $k \in \mathbb{R}$ with $f'(a) < k < f'(b)$. Then prove that there exists at least one point $c \in (a, b)$ such that $f'(c)=k$. 4

(b) If $\varphi(x)=f(x)+f(1-x)$ and $f''(x)<0$ in $[0, 1]$. Then show that φ is monotone increasing in $[0, \frac{1}{2})$ and monotone decreasing in $(\frac{1}{2}, 1]$. 4

9. (a) State Rolle's Theorem. If $p(x)$ be a polynomial of degree > 1, prove that there is a root of $p'(x)+kp(x)=0$, k being a real constant, between two distinct roots of $p(x)=0$. 2+3

(b) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as follow: 3

$$f(x)=\begin{cases} x^2 \cos\frac{1}{x}, & x \neq 0 \\ 0, & x=0 \end{cases}$$

Show that f is derivable at $x=0$ but derived function is not continuous at $x=0$.

10.(a) If a function f be such that $f^{(n)}(a)$ exists and M be defined as follows: 4

$$f(a+h)=f(a)+hf'(a)+\frac{h^2}{2!}f''(a)+\ldots+\frac{h^{n-1}}{(n-1)!}f^{(n-1)}(a)+\frac{h^n}{n!}M$$

Then show that $M \rightarrow f^{(n)}(a)$ as $h \rightarrow 0^+$.

(b) Show that for the function $f(x)=\dfrac{(2x-1)(x-8)}{x^2-5x+4}$ the minimum value is greater than the maximum value. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 3rd Semester Examination, 2022-23

MTMACOR06T-MATHEMATICS (CC6)

GROUP THEORY I

Time Allotted: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Let $\alpha=(1\ 2\ 4\ 6)$ and $\beta=(3\ 5\ 7)$ be two members of the symmetric group S_7. Find $\alpha\beta\alpha^{-1}$.

(b) Let $G=\langle a\rangle$ be a cyclic group of order 30. Find the order of the subgroup $\langle a^5\rangle$.

(c) Show that a group of order 119 can have atmost 112 elements of order 17.

(d) A binary operation $*$ on $\mathbb{Z}$ is defined by $m*n=2m+n$. Show that there is a left identity element but no right identity element.

(e) Find all the elements of order 4 in D_4, the dihedral group of order 4.

(f) Let H and K be the subgroups of a group G. Prove that the set $N_K(H)=\{x\in K : xH=Hx\}$ is a subgroup of G.

(g) Let $G=H\times K$ be the external direct product of two groups H and K. Prove that the set $S=\{(e,a): e$ is the identity of the group H and $a\in K\}$ is a normal subgroup of G.

(h) If $f=(i_1\,j_1)(i_2\,j_2)\cdots(i_k\,j_k)$ is a product of finite number of transpositions, find f^{-1}.

(i) If H and K are subgroups of a group G with $o(H)=18$ and $o(K)=35$. Find $o(H\cap K)$.

2. (a) Show that the set of all 2×2 real orthogonal matrices form a group with respect to matrix multiplication. 3

(b) Let $T=\{1,-1\}$ and $S=T\times T$. Let f and g be two bijections from S onto S defined by $f(x,y)=(x,-y)$ and $g(x,y)=(y,-x)$ for all $(x,y)\in S$. Prove that the set $G=\{f^i\circ g^i : i=0,1;\ j=0,1,2,3\}$ forms a group under the composition '$\circ$' of mappings, where $f^i=f\circ f\circ\cdots\circ f$ (i-times) and $f^0=$ the identity mapping on S. 5

3. (a) Suppose that a group G contains two elements a,b such that $o(a)=5$, $o(b)=2$ and $a^4b=ba$. Find the order of ab in G. 2

(b) In a group G, $(ab)^3=a^3b^3$ for all $a,b\in G$. Prove that the set $H=\{x^3 : x\in G\}$ is a subgroup of G. 2

(c) Let G be a group and H a nonempty finite subset of G. Prove that H is a subgroup of G if and only if $ab\in H$, for all $a,b\in H$. 4

4. (a) Let $\sigma \in S_r (r \geq 2)$ and $\sigma = \sigma_1 \sigma_2 \sigma_3 \cdots \sigma_k$ be a product of disjoint cycles in S_r. Suppose $o(\sigma_i) = n_i$, $i = 1, 2, \cdots, k$. Prove that $o(\sigma) = \text{lcm}(n_1, n_2, \cdots, n_k)$ in S_r. 4

(b) Let $\beta = (1\ 5\ 3\ 7)(9\ 6\ 8\ 4\ 2\ 10)$ in S_{10}. Find the smallest positive integer n such that $\beta^n = \beta^{-3}$. 2

(c) Let $\sigma = (1\ 3\ 7)(2\ 4\ 6\ 9)(5\ 8\ 10\ 11)$ and $\rho = (3\ 2\ 5\ 8)(4\ 7\ 10\ 1)(6\ 9\ 11)$ be two permutations in S_{11}. Find a permutation $\tau \in S_{11}$ such that $\rho = \tau\sigma\tau^{-1}$. 2

5. (a) Let H be a subgroup of a group G. For any $a \in G$, prove that the sets aH and H are equipotent. 2

(b) State and prove Lagrange's theorem for finite groups. 4

(c) Let p be a prime integer and a be an integer such that p does not divide a. Apply Lagrange's theorem to show that $a^{p-1} \equiv 1 (\text{mod}\ p)$. 2

6. (a) Prove that a finite group G of order n is cyclic if and only if it has an element of order n. 4

(b) Find all cyclic subgroup of the symmetric group S_3. 2

(c) Let G be a cyclic group of order 24 and $a \in G$. If $a^8 \neq e$ and $a^{12} \neq e$ then show that $G = \langle a \rangle$. 2

7. (a) Let $G = U_{16}$, the group of units modulo 16, $H = \{[1], [15]\}$ and $K = \{[1], [9]\}$. Find G/H, G/K and HK. 3

(b) Show that a subgroup of index 2 is a normal subgroup. 2

(c) Let $H = \langle [8] \rangle$ in $\mathbb{Z}_{24}$. What is the order of $[14] + H$ in $\mathbb{Z}_{24}$? 3

8. (a) Define kernel of a group homomorphism. Show that the kernel is a normal subgroup of the domain. 1+3

(b) Show that the function $\phi : (\mathbb{R}, +) \to (S^1, \cdot)$ defined by $\phi(x) = e^{2\pi i x}, x \in \mathbb{R}$, is a group homomorphism, where S^1 is the multiplicative group of all complex numbers z with $|z| = 1$. Find the kernel of the homomorphism ϕ. 2+2

9. (a) Let G and G' be two finite groups and $f : G \to G'$ be a group homomorphism. Show for every $a \in G$, that $o(f(a))$ divides $o(a)$. 2

(b) Find the number of group homomorphisms from the cyclic group $\mathbb{Z}_{10}$ to the cyclic group $\mathbb{Z}_{21}$. 2

(c) Prove that any group of order 6 is either isomorphic to $\mathbb{Z}_6$ or to S_3. 4

10. (a) Let H and K be two subgroups of a group G. If K is normal in G, prove that 3

$$H/(H \cap K) \simeq (HK)/K.$$

(b) Show that $\mathbb{Z}_6$ is not a homomorphic image of $\mathbb{Z}_9$. 2

(c) Let G denote the Klein's 4-group. Find a subgroup H of the symmetric group S_4 such that G is isomorphic to H. 3

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 3rd Semester Examination, 2022-23

MTMACOR07T-MATHEMATICS (CC7)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *four* questions from the rest

1. Answer any ***four*** questions from the following: $2\times4=8$

(a) If $y=4x^6-5x$, find the percentage error in y at $x=1$, if the error in $x=0.04$.

(b) What are the advantages and disadvantages of the Bisection method for finding a root of the equation $f(x)=0$.

(c) Write down Newton's forward interpolating polynomial with usual notations.

(d) For any positive number k, prove that $y_k=\sum_{i=0}^{k}\binom{k}{i}\Delta^i y_0$, Δ being the forward difference operator.

(e) Write down the formula for Weddle's rule for evaluating $\int_a^b f(x)dx$ using 12 subintervals. Is there any restriction on the number of subintervals for this particular rule?

(f) Given $\frac{dy}{dx}=x^3+y,\ y(0)=1$. Compute $y(0.02)$ by Euler's method, correct upto four decimal places taking step length 0.01.

(g) Write 'T' for True and 'F' for False statement.

(i) In Simpson's $\frac{1}{3}$rd rule $\int_{x_0}^{x_2} y\,dx=\frac{3h}{4}[y_0+4y_1+y_2]$

(ii) $\left(\frac{dy}{dx}\right)_{x=x_0}=\frac{1}{h}\left[\Delta y_0-\frac{1}{2}\Delta^2 y_0+\frac{1}{3}\Delta^3 y_0-\cdots\right]$

2. (a) The percentage error in R, which is given by $R=\frac{r^2}{2h}+\frac{h}{2}$, is not allowed to exceed 0.2%. Find allowable error in r and h when $r=4.5$ cm and $h=5.5$ cm. 4+4

(b) Perform three iterations of the Newton-Raphson method to obtain the approximate value of $(17)^{1/3}$ starting with the initial approximation $x_0=2$.

3. (a) Find $\Delta^{10}[(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)]$ 2+6

(b) Find the value of $f(x)$ for $x = 2.55$ from the following data

x	1	2	3	5
$f(x)$	3	10	29	127

4. (a) Design an algorithm to compute the HCF and LCM of two numbers, provided by user. 4+4

(b) Evaluate the integral $\int_0^5 \frac{dx}{4x+5}$ by Weddle's Rule.

5. (a) Using LU decomposition method, find the inverse of the matrix 5+3

$$\begin{bmatrix} 2 & -2 & 4 \\ 2 & 3 & 2 \\ -1 & 1 & -1 \end{bmatrix}$$

(b) From the following table, find the area bounded by the curve and x-axis from $x = 7.47$ to $x = 7.52$ by trapezoidal rule:

x	7.47	7.48	7.49	7.50	7.51	7.52
$f(x)$	1.93	1.95	1.98	2.01	2.03	2.06

6. (a) Find the largest eigen-value and the corresponding eigenvector of the following matrix by power method (correct upto 2D) 5+3

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

(b) Establish numerical differentiation formula based on Newton's forward difference formula for equispaced arguments.

7. (a) Solve the following system by Gauss Elimination method 4+4

$$x_1 + x_2 + 2x_3 = 4$$
$$x_1 + 2x_2 + 3x_3 = 6$$
$$2x_1 + 3x_2 + x_3 = 6$$

(b) Use method of successive approximation for finding approximate solution of the equation $\frac{dy}{dx} = x - y$, $y(0) = 1$.

8. Describe the power method for finding the largest (in magnitude) eigen value of a real square matrix A. How can the least eigen value (in magnitude) be obtained by using power method? Explain it mathematically. 8

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2023

MTMACOR08T-MATHEMATICS (CC8)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Let $f:[0,1]\to\mathbb{R}$ be defined by

$$f(0)=0,$$
$$f(x)=(-1)^n,\ \frac{1}{n+1}<x\le\frac{1}{n},\ n=1,2,3,\dots$$

Show that f is integrable on [0, 1].

(b) Let $f:[0,2]\to\mathbb{R}$ be a function defined by

$$f(x)=2x,\quad 0\le x\le 1$$
$$=x^2,\quad 1<x\le 2$$

Show that f has no primitive although f is integrable on [0, 2].

(c) Find the values of *p*, if any, so that the integral

$$\int_1^\infty \frac{dx}{x^p} \text{ is convergent.}$$

(d) Determine the radius of convergence of the power series

$$\sum_{n=0}^{\infty}\frac{(n+1)x^n}{(n+2)(n+3)}.$$

(e) Test the uniform convergence of the sequence of functions $\{f_n\}$ on [0, 1] defined by $f_n(x)=x^n(1-x)$, $0\le x\le 1$.

(f) Verify whether the series $\sum_{n=1}^{\infty}\frac{x}{n(n+1)}$ converges uniformly in $[0,a]$ where $a>0$.

(g) Justify true or false: The function $f(x)=\sin x$, $0\le x\le\pi$, can be expressed as a Fourier cosine series.

(h) If the power series $\sum_{n=1}^{\infty}a_nx^n$ is convergent for all $x\in\mathbb{R}$ find the value of $\limsup_{n\to\infty}|a_n|^{\frac{1}{n}}$.

2. (a) (i) Prove that a monotone function f defined on a closed interval $[a, b]$ is integrable in the sense of Riemann. 2+2

(ii) Show that the function $f:[0, n] \to \mathbb{R}$ defined by

$$f(x) = \frac{x}{[x]+1}, \quad 0 \le x \le n,$$

where $n \in \mathbb{N}$, $n > 1$, is R-integrable.

(b) If f be integrable on $[a, b]$ then show that the function F defined by 4

$$F(x) = \int_a^x f(t)\,dt, \quad x \in [a, b]$$

is continuous on $[a, b]$.

3. (a) Show that the integral 4

$\int_0^1 x^{m-1}(1-x)^{n-1}dx$ converges if and only if $m > 0$, $n > 0$.

(b) Show that the integral $\int_0^\infty \frac{x^{p-1}}{1+x}dx$ is convergent only when $0 < p < 1$. 4

4. (a) If for each $n \in \mathbb{N}$, $f_n : [a, b] \to \mathbb{R}$ be a function such that $f_n'(x)$ exists for all $x \in [a, b]$; $\{f_n(c)\}_n$ converges for some $c \in [a, b]$ and the sequence $\{f_n'\}_n$ converges uniformly in $[a, b]$, then prove that the sequence $\{f_n\}_n$ converges uniformly on $[a, b]$. 4

(b) The function f_n on $[-1, 1]$ are defined by $f_n(x) = \frac{x}{1+n^2x^2}$. Show that $\{f_n\}$ converges uniformly and that its limit function f is differentiable but the equality $f'(x) = \lim_{n\to\infty} f_n'(x)$ does not hold for all $x \in [-1, 1]$. 4

5. (a) Let g be a continuous function defined on [0, 1]. For each n in $\mathbb{N}$ define $f_n(x) = x^n g(x)$, $x \in [0, 1]$. Find a condition on g for which the sequence $\{f_n\}$ converges uniformly. 4

(b) If the series $\sum f_n$ converges uniformly in an interval $[a, b]$ prove that the sequence $\{f_n\}$ converges uniformly to the constant function 0 in $[a, b]$. 4

6. (a) Prove that $\frac{1}{2} < \int_0^1 \frac{dx}{\sqrt{4-x^2+x^3}} < \frac{\pi}{6}$. 4

(b) Show that improper integral $\int_0^\infty \frac{\sin x}{x}dx$ is convergent. 4

7. (a) Let $\sum_{n=0}^{\infty} a_n x^n$ be a given power series and $\mu = \overline{\lim} |a_n|^{1/n}$. Then show that the series is everywhere convergent if $\mu = 0$. 4

(b) Assuming $\frac{1}{1+x^2}=1-x^2+x^4-x^6+\cdots$ for $-1<x<1$, obtain the power series expansion for $\tan^{-1}x$. Also deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots=\frac{\pi}{4}$. 3+1

8. Show that the function $f:[-\pi,\pi]\to\mathbb{R}$ be defined by 2+3+1+2

$$f(x)=\begin{cases}\cos x & 0\le x\le\pi\\ -\cos x & -\pi\le x<0\end{cases}$$

satisfies Dirichlet's condition in $[-\pi,\pi]$. Obtain the Fourier co-efficients and the Fourier series for the function $f(x)$. Hence find the sum of the series

$$\frac{2}{1.3}-\frac{6}{5.7}+\frac{10}{9.11}-\cdots\cdots$$

9. (a) Let $f_n(x)=\frac{nx}{1+nx}$, $x\in[0,1]$, $n\in\mathbb{N}$. Then show that 4

$$\lim_{n\to\infty}\int_0^1 f_n(x)\,dx=\int_0^1\lim_{n\to\infty}f_n(x)\,dx,$$

but $\{f_n\}_n$ is not uniformly convergent on [0, 1].

(b) Prove that the even function $f(x)=|x|$ on $[-\pi,\pi]$ has cosine series in Fourier's form as 3+1

$$\frac{\pi}{2}-\frac{4}{\pi}\left\{\cos x+\frac{\cos 3x}{3^2}+\frac{\cos 5x}{5^2}+\cdots\cdots\right\}$$

Show that the series converges to $|x|$ in $[-\pi,\pi]$.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2023

MTMACOR09T-MATHEMATICS (CC9)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) If S be the set of all points (x, y, z) in $\mathbb{R}^3$ satisfying the inequality $x+y+z<1$, determine whether or not S is open. 2

(b) Is the set $\mathbb{R}^n$ open? Justify your answer. 2

(c) Find the closure of $\{(x, y): 1<x^2+y^2<2\}$. 2

(d) When a rational function $f(x)=\dfrac{P(x)}{Q(x)}$ (where P, Q are polynomials in the components of x) is continuous at each point x? 2

(e) Show that the function $f(x,y)=|x|+|y|$, $(x, y)\in\mathbb{R}^2$ possesses an extreme value at (0, 0) although $f_x(0,0)$, $f_y(0,0)$ do not exist. 2

(f) Find the gradient vector at each point at which it exists for the scalar field defined by $f(x, y)=x^2+y^2\sin(xy)$. 2

(g) Find $\iint\limits_R x^2dxdy$ where R is the region bounded by $x=0$, $y=0$ and $y=\cos x$. 2

(h) Use Green's theorem to compute the work done by the force field $f(x, y)=(y+3x)i+(2y-x)j$ in moving a particle once around the ellipse $4x^2+y^2=4$ in the counterclockwise. 2

2. (a) Show that the function is discontinuous at (0, 0) 4

$$f(x, y)=\begin{cases}\dfrac{x^3+y^3}{x-y}, & \text{when } x\neq y\\ 0\ , & x=y\end{cases}$$

(b) If $f(x, y)$ is continuous at (a, b) and $f(a, b)\neq 0$ then prove that there exists a neighbourhood of (a, b) where $f(x, y)$ and $f(a, b)$ maintain the same sign. 4

3. (a) The scalar field is defined by 1+1+1+1

$$f(x, y)=\begin{cases} 3y, & \text{when } x=y\\ 0\ , & \text{otherwise}\end{cases}$$

Do the partial derivatives $D_1f(0,0)$ and $D_2f(0,0)$ exist? If exist find their values. Find the directional derivative at the origin in the direction of the vector $i+j$.

(b) Evaluate $\iint\limits_R(x+2y)\,dxdy$, over the rectangle $R=[1, 2, 3, 5]$. 4

4. (a) Show that if $xyz = a^2(x+y+z)$, then the minimum value of $xy+zx+zy$ is $9a^2$. 4

(b) A function f is defined on the rectangle $R=[0,1;0,1]$ as follows: 4

$$f(x,y)=\begin{cases} \frac{1}{2}, & \text{when } y \text{ is rational} \\ x\ , & \text{when } y \text{ is irrational} \end{cases}$$

Show that the double integral $\iint_R f(x,y)\,dxdy$, does not exist.

5. (a) If $lx+my+nz=1$, l, m, n are positive constants, show that the stationary value of $xy+yz+zx$ is $(2lm+2mn+2nl-l^2-m^2-n^2)^{-1}$. 4

(b) For the vector field $F(x,y,z)=(x^2+yz)i+(y^2+xz)j+(z^2+xy)k$ compute the curl and divergence. 2+2

6. (a) Show that the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is $\frac{8abc}{3\sqrt{3}}$ (using Lagrange's method of multiplier). 4

(b) Let $y=F(x,t)$, where F is a differentiable function of two independent variables x and t which are related to two variables u and v by the relations $u=x+ct$, $v=x-ct$ $(c=\text{constant}\neq 0)$. Prove that the partial differential equation $\frac{\partial^2 y}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}=0$ can be transformed into $\frac{\partial^2 y}{\partial u \partial v}=0$. 4

7. (a) Evaluate $\iint_E (x^2+y^2)\,dxdy$ over the region E bounded by $xy=1$, $y=0$, $y=x$, $x=2$. 4

(b) Show that $\iiint_E z^2 dxdydz$, where E is the region of the hemisphere $z\geq 0$, $x^2+y^2+z^2\leq a^2$, is $\frac{2}{15}\pi a^5$. 4

8. (a) Show that the entire volume bounded by the positive side of the three co-ordinate planes and the surface $\left(\frac{x}{a}\right)^{1/2}+\left(\frac{y}{b}\right)^{1/2}+\left(\frac{z}{c}\right)^{1/2}=1$ is $\frac{abc}{90}$. 4

(b) Evaluate $\oint_C \vec{F}\cdot d\vec{r}$ around the triangle OPQ whose vertices are $O(0,0,0)$, $P(2,0,0)$ and $Q(2,1,1)$, where $\vec{F}=(2x^2+y^2)\hat{i}+(3y-4z)\hat{j}+(x-y+z)\hat{k}$. 4

9. (a) Using Stokes' theorem, evaluate $\oint_C (xydx+xy^2dy)$, where C is the square in the xy-plane with vertices (1, 0), (–1, 0), (0, 1), (0, –1). 4

(b) Using Green's theorem, evaluate $\int_\Gamma \{(y-\sin x)\,dx+\cos x\,dy\}$ where Γ is the triangle enclosed by the lines $y=0, x=\pi$ and $y=\frac{2x}{\pi}$. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2023

MTMACOR10T-MATHEMATICS (CC10)

RING THEORY AND LINEAR ALGEBRA-I

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) If in a ring $R, a^2 = a$ for all $a \in R$, prove that $a+b=0 \Rightarrow a=b$ for all $a, b \in R$.

(b) Let R be a ring with 1. Show that if R is a division ring, then R has no non-trivial ideal.

(c) Show that the characteristic of an integral domain D is either zero or a prime.

(d) Let f be a homomorphism of a ring R into a ring R'. Prove that $f(R) = \{f(a) : a \in R\}$ is a subring of R'.

(e) Let $S = \{(x, y) : x, y \in \mathbb{R}\}$. For $(x, y) \in S, (s, t) \in S$ and $c \in \mathbb{R}$, define $(x, y) + (s, t) = (x+s, y-t)$ and $c(x, y) = (cx, cy)$. Is S a vector space over $\mathbb{R}$? — Justify.

(f) Let V be a vector space of real matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and

$W = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in V : a+b=0\right\}$. Prove that W is a subspace of V.

(g) Find the dimension of the subspace S of the vector space $\mathbb{R}^3$ given by $S = \{(x, y, z) \in \mathbb{R}^3 : 2x + y - z = 0\}$.

(h) Define $T : P_n(\mathbb{R}) \to P_{n-1}(\mathbb{R})$ by $T(f(x)) = f'(x)$, where $f'(x)$ denotes the derivative of $f(x)$. Show that T is a linear transformation.

2. (a) Find all subrings of the ring $\mathbb{Z}$ of integers. 4

(b) Let R be a commutative ring with 1 and M be an ideal of R. Show that M is a maximal ideal if and only if R/M is a field. 4

3. (a) Show that $\mathbb{Z}[\sqrt{3}] = \{a + b\sqrt{3} : a, b \in \mathbb{Z}\}$ is an integral domain but not a field. 2+2

(b) Let $n \in \mathbb{Z}$ be a fixed positive integer. If $\mathbb{Z}/\langle n\rangle$ is a field, then show that n is prime, where $\langle n\rangle = \{qn : q \in \mathbb{Z}\}$ and $\mathbb{Z}/\langle n\rangle = \{a + \langle n\rangle : a \in \mathbb{Z}\}$. 4

4. (a) Prove that the cancellation law holds in a ring $(R, +, \cdot)$ if and only if $(R, +, \cdot)$ contains no divisor of zero. 4

(b) If $(R, +, \cdot)$ is an integral domain of prime characteristic p then prove that $(a+b)^p = a^p + b^p$, for all $a, b \in R$. 4

5. (a) Let A be an ideal of a ring R. Define $f : R \to R/A$ by $f(r) = r + A$, for all $r \in R$. Prove that f is a ring homomorphism. 3

(b) If f is a homomorphism of a ring R into a ring S then prove that $R/\ker f \simeq f(R)$. 5

6. (a) Let W_1, W_2 be two subspaces of a vector space V over a field $\mathbb{F}$. Prove that $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$. 4

(b) Let $W = \{(x, y, z) \in \mathbb{R}^3 : x - 4y + 3z = 0\}$. Show that W is a subspace of $\mathbb{R}^3$. Also find a basis of W. 2+2

7. (a) Let V be a vector space over a field $\mathbb{F}$, with a basis consisting of n elements. Then show that any $n+1$ elements of V are linearly dependent. 4

(b) Let V be a vector space of dimension m and W be a vector space of dimension n over a field $\mathbb{F}$. 4

Prove that $\dim(V/W) = m - n$.

8. (a) Let V and W be the vector spaces over the field F and let $T : V \to W$ be a linear transformation. If V is of finite dimension then prove that 5

$$\dim(V) = \dim(\ker T) + \dim(\operatorname{Im} T)$$

(b) Find the linear transformation $T : R^2 \to R^2$ such that $T(2, 3) = (2, 3)$ and $T(1, 0) = (0, 0)$. 3

9. (a) Let $g(x) = 3 + x$. Let $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ and $U : P_2(\mathbb{R}) \to \mathbb{R}^3$ be the linear transformations respectively defined by 4

$$T(f(x)) = f'(x)g(x) + 2f(x) \text{ and } U(a + bx + cx^2) = (a + b, c, a - b).$$

Let β and γ be the standard ordered bases for $P_2(\mathbb{R})$ and $\mathbb{R}^3$ respectively. Compute $[U]_\beta^\gamma$, $[T]_\beta$ and $[UT]_\beta^\gamma$.

(b) Determine whether the linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ defined by 4

$$T(a_1, a_2) = (3a_1 - a_2,\ a_2,\ 4a_1)$$

is invertible and justify your answer.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2022-23

MTMACOR11T-MATHEMATICS (CC11)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Form the partial differential equation by eliminating arbitrary functions from the following relation:

$$z = \phi(x+iy) + \psi(x-iy)$$

(b) Solve the following partial differential equation:

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$$

(c) Classify the partial differential equation (elliptic, parabolic, or hyperbolic)

$$\frac{\partial^2 u}{\partial x^2} - 5\frac{\partial^2 u}{\partial x \partial y} + 6\frac{\partial^2 u}{\partial y^2} = 0$$

(d) Find the order and degree of the partial differential equations:

(i) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y} = 0$

(ii) $\sqrt{1 + \dfrac{\partial^2 z}{\partial y^2}} = a\left(\dfrac{\partial z}{\partial x}\right)$

(e) Form the PDE by eliminating a, b, c from $z = a(x+y) + b(x-y) + abt + c$

(f) State whether the following statement is true or false with reason:
The PDE $x(y+z)p - y(z+x)q + z(x+y) = 0$ is quasi-linear.

(g) Prove that $pv = h$ in a central orbit, where the symbols have their usual significance.

(h) A point moves along the arc of a cycloid in such a manner that the tangent at it rotates with constant angular velocity. Show that the acceleration of the moving point is constant in magnitude.

(i) A comet describes a parabola about the Sun. Prove that the sum of the squares of its velocities at the extremities of a focal chord is constant.

2. (a) Find the integral surface given by the equation $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x+y=0$, $z=1$. 5

(b) Find a complete integral of $z = px + qy + p^2 + q^2$. 3

3. Solve by the method of separation of variables: 8

$$4\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} - 3z = 0, \text{ given that } z = 3e^{-y} - 3e^{-5y} \text{ when } x = 0.$$

4. (a) Reduce the partial differential equation $yu_x + u_y = x$ to canonical form and obtain general solution. 4

(b) Obtain the solution of the quasi linear p.d.e. $(y-u)u_x + (u-x)u_y = x - y$ with conditions $u = 0$ on $xy = 1$ using characteristic equation. 4

5. Solve the one-dimensional wave equation: 8

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0,\ t > 0$$

subject to the boundary conditions $u(0, t) = 0,\ u(L, t) = 0,\ t > 0$ and the initial conditions $u(x, 0) = f(x),\ u_t(x, 0) = g(x)$.

6. (a) Find the differential equation of all surfaces of revolution having z-axis as the axis of revolution. 4

(b) Find the characteristics of the equation 4

$$y^2 \frac{\partial^2 z}{\partial x^2} - x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

7. Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to the condition $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin\frac{n\pi}{l}x$ in $0 \le x \le l,\ 0 \le y \le a$. 8

8. A particle of mass m moves under a central attractive force $m\mu(5r^{-3} + 8c^2r^{-5})$ and it is projected from an apse at a distance c with a velocity $\frac{3\sqrt{\mu}}{c}$. Prove that the orbit is $r = c\cos\frac{2}{3}\theta$. Show further that it will arrive at the origin after a time $\frac{\pi c^2}{8\sqrt{\mu}}$. 8

9. A particle is projected with a velocity v from the Cusp of a smooth cycloid whose axis is vertical and vertex downwards, down the arc. Show that the time of reaching the vertex is 8

$$2\sqrt{\frac{a}{g}}\tan^{-1}\left(\frac{1}{v}\sqrt{4ag}\right)$$

10. The volume of a spherical raindrop falling freely increases at each instant by an amount equal to μ times its surface area at that instant. If the initial radius of the drop be 'a', then show that its radius is doubled when it has fallen through a distance $\frac{9a^2g}{32\mu^2}$. 8

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2022-23

MTMACOR12T-MATHEMATICS (CC12)

Time Allotted: 2 Hours　　　　　　　　　　　　　　　　Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Show that the function $f:\mathbb{R}^+ \to \mathbb{R}^+$ defined by $f(x)=\sqrt{x}$ for all $x\in\mathbb{R}^+$ is an automorphism of the multiplicative group of positive real numbers.

(b) Consider the elements $a=(1\ 2\ 3)$ and $b=(1\ 4)$ in S_4. Determine the commutator $[a, b]$ of a and b in S_4.

(c) Let $X=\{1, 2, 3, 4, 5\}$ and suppose that G is the permutation group defined as $\{(1), (1\ 2\ 3), (1\ 3\ 2), (4\ 5), (1\ 2\ 3)(4\ 5), (1\ 3\ 2)(4\ 5)\}$. Let X be the G-set under the action $\sigma.x=\sigma(x)$, for all $\sigma\in G$, $x\in X$. Find all the distinct orbits of X under the given action.

(d) Is there any group of order 9 whose class equation is given by $9 = 1+1+1+3+3$? Justify your answer.

(e) Show that $Z(G)$ is a characteristic subgroup of G.

(f) Let G be a group of order 125 then show that G has a non-trivial Abelian subgroup.

(g) Prove or disprove: Every group of order 76 contains a unique element of order 19.

(h) Prove that the external direct product $\mathbb{Z}_2\times\mathbb{Z}_3$ of $\mathbb{Z}_2$ and $\mathbb{Z}_3$ is isomorphic with the group $\mathbb{Z}_6$.

(i) Prove or disprove: A_4 is simple.

2. (a) Let G denote the Klein's 4-group. Find the order of the automorphism group $\text{Aut}(G)$ of G. 2

(b) Let G be a group and for each $a\in G$, $f_a: G\to G$ denote the mapping defined by $f_a(g)=gag^{-1}$ for all $g\in G$. Consider the set $\text{Inn}(G)=\{f_a : a\in G\}$. Prove that $\text{Inn}(G)$ is a normal subgroup of the automorphism group of G. 4

(c) Give examples of two non-isomorphic finite groups whose automorphism groups are isomorphic to each other. Justify your choice of groups. 2

3. (a) Show that commutator subgroup of a group G is a characteristic subgroup of G. 3

(b) Show that every characteristic subgroup is a normal subgroup but the converse need not be true. 3

(c) Let $U(n)$ denote the group of units modulo $n>1$. Express $U(144)$ as an external direct product of cyclic groups. 2

4. (a) Show that the group of all automorphisms of a finite cyclic group of order n is isomorphic to the group U_n of units modulo n. 4

(b) Determine the group of all automorphisms of the additive group of all multiples of 3. 4

5. (a) If G be a cyclic group of order mn where $\text{g.c.d}(m, n) = 1$ show that G is isomorphic to the external direct product $P \times Q$ where order of the group P is m and order of the group Q is n. 4

(b) Determine the number of elements of order 5 in $\mathbb{Z}_{25} \times \mathbb{Z}_5$, the external direct product of the groups $\mathbb{Z}_{25}$ and $\mathbb{Z}_5$. 4

6. (a) State fundamental theorem of finite abelian groups. 2

(b) Describe all the abelian groups of order 539. Hence show that every such abelian group has an element of order 77. 4+2

7. (a) Let G be a finite group of order 847 and H be a subgroup of G of index 7. Apply generalized Cayley's theorem to show that H is a normal subgroup of G. 4

(b) Find the number of distinct conjugacy classes of the symmetric group S_5. Determine the order of the conjugacy class of the permutation $\alpha = (1\ 2)(3\ 4)$ in S_5. 1+3

8. (a) Let G be a group of permutations of a set S. For each $s \in S$ define stabilizer of S in G and orbit of s under G. 1+1+4
Show that, for any finite group of permutations of a set S,

$$|G| = |\text{orb}_G(s)|\,|\text{stab}_G(s)| \quad \forall\, s \in S.$$

(b) Let $G = \{(1),\ (1\ 2\ 3)(4\ 5\ 6)(7\ 8),\ (1\ 2\ 3)(4\ 5\ 6)(1\ 3\ 2)(4\ 6\ 5),$ $(1\ 3\ 2)(4\ 6\ 5)(7\ 8)\}$ 1+1
Find $\text{orb}_G(4)$ and $\text{stab}_G(4)$.

9. (a) Let G be a finite group of order $p^n m$, where p is a prime integer, n is a non-negative integer and m is a positive integer such that p does not divide m. If n_p denotes the number of Sylow p-subgroups of G, prove the following assertions: 3+2

(i) $n_p \equiv 1 \pmod{p}$, (ii) n_p devides $|G|$.

(b) Let G be a group of order 99. If G has a normal subgroup of order 9, show that G is a commutative group. 3

10.(a) Let G_1 and G_2 be two groups. Prove that the direct product $G_1 \times G_2$ is commutative if and only if both G_1 and G_2 are commutative. 2

(b) Show that the direct product $Z_6 \times Z_4$ of the cyclic groups Z_6 and Z_4 is not a cyclic group. 3

(c) Find all Abelian groups of order 63 which contain an element of order 21. 3

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2022-23

MTMADSE01T-MATHEMATICS (DSE1/2)

LINEAR PROGRAMMING

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Prove that the vectors $(1,1,0)$, $(0,1,1)$ and $(1,2,1)$ form a basis in E^3.

(b) Check whether $x=5$, $y=0$, $z=-1$ is a basic solution of the system of equations:

$$x+2y+z=4,$$
$$2x+y+5z=5$$

(c) If $C(X)=\{(x,y):|x|\le 2,|y|\le 1\}$ be a convex hull then find set X.

(d) Find graphically the feasible space, if any, of the following:

$$x_1+2x_2\ge 2$$
$$5x_1+3x_2\le 15,\ x_1,x_2\ge 0$$

(e) Define fair game and strictly determinate game.

(f) Find the maximum number of possible way of assignment of a 5×5 assignment problem.

(g) What is the criterion for no feasible solution in two-phase method?

(h) Define saddle point. Find the value of the game of the pay-off matrix

Player Q

Player P		B_1	B_2
	A_1	1	-1
	A_2	-1	1

2. A business manager has the option of investing money in two plans. Plan A guarantees that each rupee invested will earn 70 paise a year and plan B guarantees that each rupee invested will earn Rs. 2.00 every two years. In plan B, only investments for periods that are multiples of 2 years are allowed. How should the manager invest Rs. 50,000/- to maximize the earnings at the end of 3 years? Formulate the problem as a Linear Programming Problem with two legitimate variable. Find the optimum solution using graphical method. 4+4

3. State and prove fundamental theorem of LPP. 8

4. Use Two Phase method to solve the following linear programming problem: 8

Maximize $z = 2x_1 + x_2 + x_3$

Subject to $4x_1 + 6x_2 + 3x_3 \leq 8$

$3x_1 - 6x_2 - 4x_3 \leq 1$

$2x_1 + 3x_2 - 5x_3 \geq 4$

$x_1, x_2, x_3 \geq 0$

5. (a) Prove that the set of all convex combination of a finite number of points is a convex. 4

(b) Reduce the feasible solution $(1, 2, 1)$ of the following system of equation to a basic feasible solution. 4

$x_1 - x_2 + 2x_3 = 1$

$x_1 + 2x_2 - x_3 = 4$

6. State and prove fundamental theorem of duality. 8

7. Solve the following LPP using duality theory: 8

Minimize $z = x_1 + x_2 + x_3$

Subject to $x_1 - 3x_2 + 4x_3 = 5$

$x_1 - 2x_2 \leq 3$

$2x_2 - x_3 \geq 4$

$x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

8. (a) Find the optimal assignment and the corresponding assignment cost for the assignment problem with the following cost matrix: 4

	D_1	D_2	D_3	D_4	D_5
O_1	2	4	3	5	4
O_2	7	4	6	8	4
O_3	2	9	8	10	4
O_4	8	6	12	7	4
O_5	2	8	5	8	8

(b) Find the initial B.F.S. of the following transportation problem by VAM method hence find the optimal solution: 4

	D_1	D_2	D_3	D_4	a_j
O_1	19	14	23	11	11
O_2	15	16	12	21	13
O_3	30	25	16	39	18
b_j	6	10	11	15	

9. Prove that the mixed strategies p^*, q^* will be optimal strategy of the game if and only if $E(p, q^*) \leq E(p^*, q^*) \leq E(p^*, q)$ 8

10.(a) Solve graphically the following game problem: 5

A \ B	B_1	B_2
A_1	2	7
A_2	3	5
A_3	11	2

(b) Use dominance method to reduce the payoff matrix in a 2×2 game. Hence solve it. 3

	B_1	B_2	B_3
A_1	8	5	8
A_2	8	6	5
A_3	7	4	5
A_4	6	5	6

11. In a rectangular game, the pay-off matrix is given by 8

Player P \ Player Q	Q_1	Q_2	Q_3
P_1	3	2	–1
P_2	4	0	5
P_3	–1	3	–2

State with justification, whether the players will choose pure or mixed strategies. Solve the game problem by converting it into a L.P.P.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2022-23

MTMADSE03T-MATHEMATICS (DSE1/2)

PROBABILITY AND STATISTICS

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Define a random experiment and event space.

(b) Consider events A and B such that $P(A)=\frac{1}{4}$, $P(B\,|\,A)=\frac{1}{2}$, $P(A\,|\,B)=\frac{1}{4}$. Find $P(\overline{A}\,|\,\overline{B})$

(c) Consider an experiment of rolling two dice. Let A be the event 'total is odd' and B be the event '6 on the first die'. Are A and B independent? Justify your answer.

(d) If $F(x)$ be the distribution function of a random variable X, then prove that $F(a)-\lim\limits_{x\to a-0} F(x)=P(X=a)$

(e) The probability density function of a random variable X is $2x\cdot e^{-x^2}$ for $x>0$ and zero otherwise. Find the probability density of X^2.

(f) The joint probability density function of random variables X and Y is given by

$$f(x,y)=\begin{cases}2(x+y-3xy^2), & 0<x<1,\quad 0<y<1\\ 0, & \text{elsewhere}\end{cases}$$

Find the marginal density functions of X and Y.

(g) Prove that $-1\le \rho(X,Y)\le 1$, the symbols having usual meaning.

(h) Find the characteristic function of a binomial $(n,\,p)$ variate.

(i) Find the mean of a Poisson μ-variate.

2. (a) If A and B are two events such that $P(A)=P(B)=1$, then show that $P(A+B)=1$, $P(AB)=1$. 3

(b) A secretary writes four letters and the corresponding addresses on envelopes. If he inserts the letters in the envelopes at random irrespective of address, then calculate the probability that all the letters are wrongly placed. 5

3. Prove that the function $f(x)$ of a random variable X defined by $f(x)=\frac{1}{2}e^{-|x|}$, $-\infty<x<\infty$, is a possible probability density function and find the corresponding distribution function and the moment generating function and hence evaluate mean and variance. 8

4. (a) Define Poisson distribution. Prove that the sum of two independent Poisson variates having parameters μ_1 and μ_2 is a Poisson variates having parameter $\mu_1+\mu_2$. 4

(b) If θ be the acute angle between two regression lines, then prove that 4

$$\tan\theta=\frac{1-\rho^2}{\rho}\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$$

where σ_x and σ_y are standard deviations of the random variables X and Y respectively. What happens when $\rho=1$?

5. (a) For the binomial (n, p) distribution, prove that $\mu_{r+1}=p(1-p)\left[nr\mu_{r-1}+\frac{d\mu_r}{dp}\right]$ where μ_r is the rth central moment of the distribution. 5

(b) If $ax+by+c=0$ be the relation between x and y, find r_{xy}. 3

6. (a) The joint probability density function of two random variables X and Y is 5

$f(x, y)=8xy,\ 0\le x\le 1,\ 0\le y\le 1$

$=0$, elsewhere.

Examine whether X and Y are independent. Also find the conditional probability density functions.

(b) Use Tchebycheff's inequality to show that for $n\ge 36$, the probability that in n throws of a fair die the number of sixes lies between $\frac{n}{6}-\sqrt{n}$ and $\frac{n}{6}+\sqrt{n}$ is at least $31/36$. 3

7. (a) Obtain the maximum likelihood estimate of θ on the basis of a random sample of size n drawn from a population whose probability density function is 5

$$f(x)=ce^{-x/\theta},\ 0\le x<\infty,$$

where c is constant and $\theta>0$.

(b) Two random variables X, Y have the least square regression lines with equations $3x+2y-26=0$ and $6x+y-31=0$. Find $E(X), E(Y)$ and $\rho(X, Y)$. 3

8. (a) A random variable X has probability density function $12x^2(1-x),\ 0<x<1$. Compute $P(|X-m|\ge 2\sigma)$ and compare it with the limit given by Tchebycheff's inequality, where m is the mean and σ is the standard deviation of the distribution. 5

(b) State and prove the law of large numbers. 3

9. (a) Find the sampling distribution of the statistic $Y=\frac{nS^2}{\sigma^2}$, where σ^2 is the population variance and $S^2=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$. 5

(b) Prove that the sample variance is a consistent estimate of the population variance but it is not an unbiased estimate of population variance. 3

10.(a) If $\{X_n\}_n$ is a sequence of independent variables such that each X_i has the same distribution with mean m and standard deviation σ, then show that $\frac{\bar{X}-m}{\sigma/\sqrt{n}}$ is asymptotically normal $(0,1)$, where $\bar{X}=\frac{X_1+X_2+\cdots+X_n}{n}$. 4

(b) A point P is chosen at random on a circle of radius a and A be a fixed point on the circle. Find the expectation of the distance AP. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 1st Semester Examination, 2022-23

MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)

DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question Number 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Examine whether the limit $\lim\limits_{x\to 3}\frac{[x]}{x}$ exists, where $[x]$ represents the greatest integer less or equal to x.

(b) $f(x)=\begin{cases} x+1 & \text{when } x\le 1 \\ 3-ax & \text{when } x>1 \end{cases}$

For what value of a, will f be continuous at $x=1$.

(c) For the function $f(x)=|x|;\ x\in\mathbb{R}$ show that $f'(0)$ does not exists.

(d) Show that the function $f(x)=4x^2-6x-11$ is increasing at $x=4$.

(e) Find the point on the curve $y=x^3-6x+7$ where the tangent is parallel to the straight line $y=6x+1$.

(f) Find the asymptotes of the curve $xy^2-yx^2-(x+y+1)=0$.

(g) Examine the continuity of the function at $(0, 0)$

$$f(x, y)=\begin{cases} \dfrac{xy}{x^2+y^2}, & (x, y)\neq(0, 0) \\ 0, & (x, y)=(0, 0) \end{cases}$$

(h) Show that the function $f(x, y)=\dfrac{x^{1/4}+y^{1/4}}{x^{1/5}+y^{1/5}}$ is homogeneous in x and y. Find its degree.

(i) If $u=x\log y$, then show that $u_{xy}=u_{yx}$.

2. (a) If f is an even function and $f'(0)$ exists, then show that $f'(0)=0$. 4

(b) Discuss the continuity of f at $x=1$ and $x=2$ where $f(x)=|x-1|+|x-2|$. 4

3. (a) If $x+y=e^{x-y}$, show that $\dfrac{d^2y}{dx^2}=\dfrac{4(x+y)}{(x+y+1)^3}$ 4

(b) State and prove Lagrange's Mean Value Theorem. 4

4. (a) Find the slope of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ at the point (x_1, y_1) and hence obtain the equation of the tangent at that point. 4

(b) Verify Rolle's theorem for the function $f(x) = x\sqrt{4-x^2}$ is $0 \le x \le 2$. 4

5. (a) Expand $f(x) = \sin x$ as a series of infinite terms. 5

(b) If $y = \dfrac{x}{x+1}$, show that $y_5(0) = 51$. 3

6. (a) If $f(x) = \log \dfrac{\sqrt{a+bx} - \sqrt{a-bx}}{\sqrt{a+bx} + \sqrt{a-bx}}$, find for what values of x, $\dfrac{1}{f'(x)} = 0$. 4

(b) Prove that $\lim\limits_{h \to 0} \dfrac{f(a+h) - 2f(a) + f(a-h)}{h^2} = f''(a)$, provided that $f''(x)$ is continuous. 4

7. (a) Find the maxima and minima, if any, of $\dfrac{x^4}{(x-1)(x-3)^3}$. 2+2

(b) Determine the values of a, b, c so that $\dfrac{a\sin x - bx + cx^2 + x^3}{2x^2 \log(1+x) - 2x^3 + x^4}$ may tend to a finite limit as $x \to 0$, and determine this limit. 3+1

8. (a) If $lx + my = 1$ is a normal to the parabola $y^2 = 4ax$, then show that $al^3 + 2alm^2 = m^2$. 4

(b) If the tangent at (x_1, y_1) to the curve $x^3 + y^3 = a^3$ meets the curve again in (x_2, y_2), show that $\dfrac{x_2}{x_1} + \dfrac{y_2}{y_1} = -1$. 4

9. (a) Prove that the asymptotes of the curve $x^2y^2 = a^2(x^2 + y^2)$ form a square of side $2a$. 4

(b) Show that for an ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, the radius of curvature at an extremity of the major axis is equal to the half of the latus rectum. 4

10.(a) If V is a function r alone, where $r^2 = x^2 + y^2 + z^2$, show that 4

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{d^2 V}{dr^2} + \frac{2}{r}\frac{dV}{dr}.$$

(b) If $y = f(x+ct) + \phi(x-ct)$, show that $\dfrac{\partial^2 y}{\partial t^2} = c^2 \dfrac{\partial^2 y}{\partial x^2}$. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2023

MTMHGEC02T/MTMGCOR02T-MATHEMATICS (GE2/DSC2)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5=10$

(a) Verify the integrability of the following differential equation

$$3x^2dx+3y^2dy-(x^3+y^3+e^{2z})dz=0.$$

(b) Find an integrating factor of the differential equation $x\dfrac{dy}{dx}=x^2+3y,\ x>0$

(c) Construct a differential equation by elimination of the arbitrary constants a and b from the equation $z=ae^{-b^2t}\cos bx$.

(d) Show that x, x^2 and x^4 are linearly independent solutions of

$$x^3\frac{d^3y}{dx^3}-4x^2\frac{d^2y}{dx^2}+8x\frac{dy}{dx}-8y=0.$$

(e) Show that the differential equation satisfied by the family of curves given by $c^2+2cy-x^2+1=0$, where c is the parameter of the family, is

$$(1-x^2)p^2+2xyp+x^2=0,\text{ where } p=\frac{dy}{dx}.$$

(f) Find the transformation of the differential equation

$$x^2\frac{d^2y}{dx^2}-5y=\log x$$

by using the substitution $x=e^z$.

(g) Determine the order, degree and linearity of the following P.D.E.

$$x\left(\frac{\partial z}{\partial x}\right)^2+xz\frac{\partial^2 z}{\partial x^2}-z\frac{\partial z}{\partial x}=0$$

(h) Form the partial differential equation by eliminating the arbitrary function from the following equation

$$z=F(x^2+y^2)$$

2. (a) Find the value of λ, for the differential equation $(xy^2+\lambda x^2y)dx+(x+y)x^2dy=0$ is exact. Solve the equation for this value of λ. 4

(b) Obtain the solution of the differential equation 4

$$xdy-ydx+a(x^2+y^2)dx=0$$

3. (a) Reduce the equation $(x^2y^3+2xy)dy=dx$ to a linear equation and solve it. 4

(b) If $y_1=e^{-x}\cos x$, $y_2=e^{-x}\sin x$ and $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$, then 2+1+1

(i) Calculate Wronskian determinant.

(ii) Verify that y_1 and y_2 satisfy the given differential equation.

(iii) Apply Wronskian test to check that y_1, y_2 are linearly independent.

4. (a) Solve $x\cos\left(\frac{y}{x}\right)(ydx+xdy)=y\sin\left(\frac{y}{x}\right)(xdy-ydx)$. 4

(b) Using the transformation $y^2=Y$, $x=X$ to solve the equation $y=2px-p^2y$ where $p=\frac{dy}{dx}$. 4

5. (a) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}+y=\tan x$ 4

(b) Solve $\frac{dx}{dt}+5x+y=e^t$, $\frac{dy}{dt}+x+5y=e^{5t}$ 4

6. (a) Solve: $(D^3-2D^2-5D+6)y=(e^{2x}+3)^2+e^{3x}\cosh x$ where $D\equiv\frac{d}{dx}$. 4

(b) Reduce the equation $(px^2+y^2)(px+y)=(p+1)^2$ to Clairaut's form by substitutions $u=xy$, $v=x+y$, where $p=\frac{dy}{dx}$. Hence find its complete solution. 4

7. (a) Find the PDE of all sphere whose centre lie on z-axis and given by equations $x^2+y^2+(z-a)^2=b^2$, a and b being constants. 4

(b) Solve: $(x^2-yz)p+(y^2-zx)q=z^2-xy$ 4

8. (a) Solve $px+qy=pq$ by Charpit's method. 4

(b) Solve: $\frac{\partial^3 z}{\partial x^3}-3\frac{\partial^3 z}{\partial x^2\partial y}+4\frac{\partial^3 z}{\partial y^3}=e^{x+2y}$ 4

9. (a) Solve the P.D.E. $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=\sin x$. 4

(b) Determine the points (x, y) at which the partial differential equation 4

$$(x^2-1)\frac{\partial^2 z}{\partial x^2}+2y\frac{\partial^2 z}{\partial x\partial y}-\frac{\partial^2 z}{\partial y^2}=0$$ is hyperbolic or parabolic or elliptic.

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लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 4th Semester Examination, 2023

MTMHGEC04T/MTMGCOR04T-MATHEMATICS (GE4/DSC4)

ALGEBRA

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Define a partial order relation. Give an example.

(b) Find the number of elements of order 5 in $\mathbb{Z}_{20}$.

(c) Find all cyclic sub-groups of the group $\{1, i, -1, -i\}$ with respect to multiplication.

(d) Prove or disprove "union of two sub-groups of a group $(G, \circ)$ is a sub-group of $(G, \circ)$".

(e) If G is a group and $a^2 = e$, $\forall a \neq e$. Prove that G is an abelian group.

(f) Is symmetric group S_3 cyclic? Give reasons.

(g) Examine whether $5\mathbb{Z} = \{5x : x \in \mathbb{Z}\}$ is an ideal or not of the ring $(\mathbb{Z}, +, \cdot)$, where $\mathbb{Z}$ is the set of all integers.

(h) Examine if the ring of matrices $\left\{\begin{pmatrix} a & b \\ 2b & 2b \end{pmatrix} : a, b \in \mathbf{R}\right\}$ contains any divisor of zero.

2. (a) Let a relation R defined on the set $\mathbb{Z}$ by " $a\, R\, b$ if and only if $a-b$ is divisible by 5 for all $a, b \in \mathbb{Z}$. Show that R is an equivalence relation. 4

(b) Show that the set of all permutations on the set $\{1, 2, 3\}$ forms a non abelian group. 4

3. (a) Show that a non-empty subset H of G forms a subgroup of $(G, \circ)$ if and only if (i) $a \in H$, $b \in H \Rightarrow a \circ b \in H$, and $a \in H \Rightarrow a^{-1} \in H$. 4

(b) Prove that $\mathrm{SL}(n, \mathbb{R})$ is a normal subgroup of $\mathrm{GL}(n, \mathbb{R})$. 4

4. (a) Show that every proper sub-group of a group of order 6 is cyclic. 4

(b) Prove that the commutator sub-group of any group is a normal sub-group. 4

5. (a) Prove that the order of every subgroup of a finite group G is a divisor of the order of G. 4

(b) Prove that quotient group of an abelian group is abelian. Is the converse true? Justify. 4

6. (a) Prove that for any positive integer n, the set $U(n) = \{[x] : x$ is positive integer less than n and prime to $n\}$ is a group with respect to 'Multiplication Modulo n'. 4

(b) Let $f : \mathbb{R} \longrightarrow \mathbb{R}^+$ be defined by $f(x) = e^x$, $x \in \mathbb{R}$, where $\mathbb{R}$ is the set of real numbers. Prove that f is invertible and find f^{-1}. 4

7. (a) Prove that any finite subgroup of the group of non zero complex numbers under multiplication is a cyclic group. 4

(b) Let $G = S_3$ be a group and $H = \{\rho_0, \rho_1, \rho_2\}$ be a subgroup of G. Find all the left cosets of H (where the symbols have their usual meanings). 4

8. (a) Show that the ring of matrices $\left\{\begin{pmatrix} 2a & 0 \\ 0 & 2b \end{pmatrix} : a, b \in \mathbf{Z}\right\}$ contains divisors of zeros and does not contain the unity. 4

(b) Let $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Z}$ (the set of integers)$\}$. Show that $(\mathbb{Z}[\sqrt{2}], +, \cdot)$ is an integral domain. 4

9. (a) Prove that the ring of matrices $\left\{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbf{R}\right\}$ is a field. 4

(b) Prove that a field is an integral domain. 4

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